DOA Estimation Method for Wideband Signal using Nested Antenna Array Based on Matrix Pencil Algorithm

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Abstract

Radio Direction Finding (RDF) systems have a lot of applications in both civil and military area such as Radio Navigation, Electronic warfare or Emergency Aid and intelligent operations. In modern communication system, wideband signals are widely used especially in wireless localization field. In this paper, a novel RDF system using Nested Antenna Array and Total Forward Backward Matrix Pencil Algorithm is proposed. This system can calculate the DOA of signals coming from the number of sources more than the number of antenna elements. The simulation results for DOA estimation of wideband signals using the proposed system will be shown and analyzed to verify its performance.

Keywords: Direction of Arrival (DOA), Nested Antenna Array (NAA), TFBMP.

1. Introduction

Direction Of Arrival (DOA) of incoming signal is the most important information estimated by Radio Direction Finding systems which have a lot of applications in practice such as Radio Navigation, Emergency Aid and intelligent operations, etc... Recently, wideband or ultra-wideband signals are widely used in wireless localization system in both civil and military areas such as radar, sonar or car collision warning system, Wireless Sensor Network [1-3]… To estimate the DOA of wideband signal, many methods have been proposed [4-7]. They can be divided into three orientations: system architecture development and research on DOA estimation algorithms or hybrid of the two.

Uniform Linear Antenna Array (ULA) model can be described as a set of M isotropic antenna elements spaced at a uniform interval d along some line in space. This is one of the most convenient mathematical models for array processing especially in Radio Direction Finding systems due to its simplicity and regularity. However, with ULA model, the number of radio incoming signals which can be detected and estimated the DOA information by RDF system always must be less than M. In order to overcome this restriction, in [8], the author proposed a novel array structure called Nested Antenna Array (NAA). This is a variant of an ULA model which can increase degrees of freedom by vectorizing the covariance matrix of the received signals at each antenna element.

In [9-10], Matrix Pencil (MP) algorithm was proposed as a high - resolution technique for DOA estimation. In this algorithm, the independent data samples are directly processed. This fact helps the MP to be less processing power and faster executing in comparison with the other super - resolution methods for DOA estimation which generally must calculate the signal covariance matrix such as MUSIC [4], ESPRIT [7]. One of the most remarkable advantages of this technique is that it can extract the DOA information with one snapshot.

The Total Forward - Backward Matrix Pencil (TFBMP) algorithm is an extension of the Matrix Pencil Method. The Total Forward - Backward is the pre - processing technique to break the correlative property of the received signals. Therefore, the DOA information of the coherent incoming signals can be accurately calculated [11]. In [12], TFBMP was used for the high - resolution frequency estimator with the better estimation results than the other methods such as Fourier technique.

In this paper, a novel method to estimate the DOA of ultra - wideband incoming signals using NAA based on TFBMP algorithm is investigated. The performance of this method will be assessed in many cases that depend on the characteristics of incoming signals as well as antenna array properties.

The paper is organized as follows. Section II describes the structure of the NAA. In section III, we present in detail the ultra-wideband signals model and TFBMP technique for DOAs of those signals. The

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simulation results are shown in the section IV. The conclusion is given in the section V.

2. Nested Antenna Array Architecture

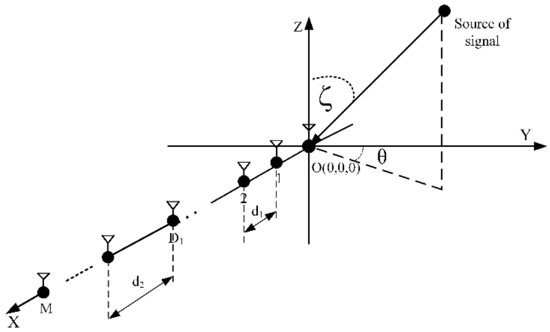

Fig. 1. Nested Antenna array in the coordinate system

In our research, we utilize an D - element Nested Antenna Array (NAA) which is a variant of ULA. Basically, NAA is composed by two ULAs that are hooked together. Two ULAs are called inner and outer array, respectively, in which the inner ULA includes D_1 antenna elements with spacing d_1 and outer ULA has D_2 elements with spacing $d_2 = (D_1 + 1)d_1$. The reference point is defined as the origin of the three - dimensional Cartesian coordinate system shown in Fig.1.

Assume that the incoming signal at the far field of the array impinging on the ULA has DOA information in both elevation (ξ) and azimuth (θ) as shown in Fig.1. However, in this work, only the signal in the same plane with antenna array is concerned. This means that the DOA of signal of interest is estimated in azimuth and $(\xi) = 90^o$.

In practice, there are several radio signals crossing the antenna array simultaneously. The received signal at each antenna element will be the sum of all arriving radio signals. In case of K signals approaching the array from some azimuth directions $\theta_1, \theta_2 \dots \theta_K$, according to [7-9], the wideband signal received at the g^{th} antenna element can be modeled as

$$x_g(t) = \sum_{i=1}^{K} s_i\left(t - v_g.sin\theta_i\right) + \varepsilon_g(t) \qquad (1)$$

where $s_i(t)$ is the i^{th} incoming signal; $\varepsilon_g(t)$ is noise at the g^{th} antenna element, which is assumed to be uncorrelated with the signal sources and is white noise in both temporal and spatial domain; $v_g = \frac{d_g}{c}$, in which d_g is the distance between the g^{th} element and the reference point, and c is the speed of the signal propagation.

Assuming the array manifolds of different DOAs are independent. In other words, K array manifolds with K different DOAs should span a K −dimensional subspace. Moreover, considering the number of signal sources is either known or can be estimated. The bandwidths of the wideband sources need not be identical, but there should be some frequency band $[\omega_L, \omega_H]$ where ω_L and ω_H are minimum and maximum angular frequency of wideband signal spectrum, respectively. In order to ensure the Fourier transform of the output signal at each antenna element has a good resolution, we suppose the observation time is long enough. Then the DFT of the m^{th} element output is

$$X_g(\omega) = \sum_{i=1}^{K} S_i(\omega) e^{-j\omega v_g sin\theta_i} + \varepsilon_g(\omega) \qquad (2)$$

Equation (2) describes the received wideband signals at each antenna element in frequency domain. In order to estimate the DOA information, this signal is split into several narrowband bins using filter banks or the DFT technique. If the intersection of the frequency bands of all incoming signals is $[\omega_L, \omega_H]$, then the output of the filter bank or DFT module can be written in frequency vector form as follows:

$$\boldsymbol{X}(\omega_l) = A(\omega_l, \theta)\boldsymbol{S}(\omega_l) + \varepsilon(\omega_l), \quad l = 0, 1 \dots N-1 \qquad (3)$$

where N is number of bins and

$$\boldsymbol{X}(\omega_l) = [\boldsymbol{X_0}(\omega_l)\boldsymbol{X_1}(\omega_l) \quad \dots \quad \boldsymbol{X_{M-1}}(\omega_l)]^T \qquad (4)$$

$$\boldsymbol{S}(\omega_l) = [\boldsymbol{S_0}(\omega_l)\boldsymbol{S_1}(\omega_l) \quad \dots \quad \boldsymbol{S_{K-1}}(\omega_l)]^T, \qquad (5)$$

in which "T" denotes transpose matrix, $\omega_L < \omega_l < \omega_H$ with $l = 0,1, \dots N-1$, $A(\omega_l, \theta)$ is the $D \times K$ steering matrix:

$$\mathbf{A}(\omega_l, \theta) = [a(\omega_l, \theta_0) \;\; a(\omega_l, \theta_1) \;\; \dots \;\; a(\omega_l, \theta_{K-1})] \qquad (6)$$

The columns of the matrix $\boldsymbol{A}$ are the $D \times 1$ array manifolds $a(\omega_l, \theta_k)$ at frequency ω_l. The array manifold is defined as

$$a(\omega_l, \theta_k) = \left[1 \;\; e^{-j\omega_l v_1 sin\theta_k} \;\; \dots \;\; e^{-j\omega_l v_{D-1} sin\theta_k}\right]^T \qquad (7)$$

where $k = 0,1, \dots, K$ and θ_k is the DOA of the k^{th} incoming signal.

3. DOA estimation based on TFBMP

In case of ULA model, the array manifold as in Eq.7 is Vandemonde in form. The DOA of incoming signal can be estimated by a lot of methods. However, in the NAA model, the form of manifold vector does not have Vandemonde form due to the varying distance between antenna elements ($d_1 \neq d_2$).

Therefore, the DOA information cannot be directly calculated using TFBMP [13]. In order to do that, the vector manifold as in Eq.7 have to be transformed into basic Vandemonde form using Kronecker (KR) product [14]. By using this product, a new full rank matrix is constructed as

$$\boldsymbol{A}_{NA} = \boldsymbol{A}^* \odot \boldsymbol{A} \tag{8}$$

where $\odot$ denotes the KR product, $\boldsymbol{A}^*$ is complex conjugate of $\boldsymbol{A}$.

$\boldsymbol{A}_{NA}$ can be considered as a steering matrix of a virtual antenna array created from NAA. This array is similar to the ULA where the number of elements is

$$M = 2D_2(D_1 + 1) - 1 \tag{9}$$

The position of each element is defined as

$$d_m = Md_1, m = -M \div M \tag{10}$$

Therefore, instead of using $\boldsymbol{A}$, $\boldsymbol{A}_{NA}$ will be used to determine the DOA information. After applying Eq.8 to Eq.6, the Eq.7 has been transformed in to Vandemonde form. In l^{th} bin, let $z_i = e^{-j\omega\frac{d}{c}sin\theta_i}$, Equation (2) can be rewritten as

$$X_m(\omega) = \sum_{i=1}^{K} S_i(\omega)z_i^m + N_m(\omega) \tag{11}$$

According to [15,18], the DOA information θ_i could be extracted by using TFBMP with the following steps.

Step 1 - Compute the Hankel matrix of $\boldsymbol{X}_m(\omega)$ - $\boldsymbol{Y}_m$

$$Y_m = \begin{bmatrix} X_0 & X_1 & \dots & X_L \\ X_1 & X_2 & \dots & X_{L+1} \\ \dots & \dots & \dots & \dots \\ X_{M-L-1} & X_L & \dots & X_{M-1} \end{bmatrix}_{(M-L)x(L+1)} \tag{12}$$

where L is the Pencil parameter. Because of the efficient noise filtering issue described in [10], L is chosen with the conditions as:

$$K \leq L \leq M - K, \text{ if } M \text{ is even.}$$
$$K \leq \mathrm{L} \leq \mathrm{M} - \mathrm{K} + 1, \text{ if } M \text{ is odd} \tag{13}$$

Step 2 - Compute all data matrix - $\boldsymbol{Y}_l$:

$$\boldsymbol{Y}_l = \begin{bmatrix} \boldsymbol{Y}_m \\ \boldsymbol{Y}_b \end{bmatrix}_{(2M-2L)\times(L+1)} \tag{14}$$

where $\boldsymbol{Y}_\mathrm{b}$ is complex conjugate matrix of $\boldsymbol{Y}_m$.

In this step, $\boldsymbol{Y}_l$ is performed SVD to obtain signal and noise subspace - $\boldsymbol{Y}_n$ and $\boldsymbol{Y}_s$, respectively.

$$\boldsymbol{Y}_\mathrm{l} = \boldsymbol{Y}_\mathrm{n} + \boldsymbol{Y}_\mathrm{s} \tag{15}$$

Step 3 - Decompose all data matrix - $\boldsymbol{Y}_l$: based on Eq. 15 and SVD operation, $\boldsymbol{Y}_l$ can be represented as follow:

$$\boldsymbol{Y}_l = \boldsymbol{U}_n\boldsymbol{\Sigma}_n\boldsymbol{V}_n^H + \boldsymbol{U}_s\boldsymbol{\Sigma}_s\boldsymbol{V}_s^H \tag{16}$$

where $\boldsymbol{U}_n$ and $\boldsymbol{V}_n$ are $(T-K)\times T$ matrices, $T = min\,((2M-2L), L+1$; $\boldsymbol{U}_s$ and $\boldsymbol{V}_s$ are $K \times T$ matrix; $\boldsymbol{U}_n$ and $\boldsymbol{U}_s$ are obtained from $\boldsymbol{U}$.

Step 4 - Extract DOA information: in order to get DOA information, z_i must be calculated. It can be extracted from matrix $\boldsymbol{U}_s$. By deleting the last and the first L columns from $\boldsymbol{U}_s$, two matrices $\boldsymbol{U}_{s1}$ and $\boldsymbol{U}_{s2}$ are created, respectively. After that, the matrix $\boldsymbol{U}_{s1}^+ \boldsymbol{U}_{s2}$ will be established, in which $\boldsymbol{U}_{s1}^+$ is Moore - Penrose pseudo - inverse of $\boldsymbol{U}_{s1}$ as

$$\boldsymbol{U}_{s1}^+ = (\boldsymbol{U}_{s1}^H\, \boldsymbol{U}_{s1})^{-1}\, \boldsymbol{U}_{s1}^H \tag{17}$$

$\boldsymbol{U}_{s1}^+\boldsymbol{U}_{s2}$ is a $K \times K$ matrix. This matrix has the eigenvalues which is the value of z_i. Therefore, by using the values of the generalized eigenvalues of $\boldsymbol{U}_{s1}^+\boldsymbol{U}_{s2}$, angles of arrival can be estimated as

$$\theta_i = arc\,sin\left[\left(-\frac{c}{2\pi f d}\right)\Im\big(ln(z_i)\big)\right] \tag{18}$$

where $\Im(ln(z_i))$ is the imaginary part of $ln(z_i)$.

Table 1. The DOAs (Degrees) estimated in each narrow bin

Bin 1	Bin 2	Bin 3	Bin 4	Bin 5	Bin 6	Bin 7	Bin 8	Bin 9	Bin 10	Bin 11
-50.0178	-50.0081	-49.8702	-49.8298	-49.7394	-50.2243	-50.1369	-49.7431	-49.7544	-50.2252	-50.2467
-20.0797	-19.8219	-20.1454	-19.8356	-19.8324	-20.3086	-20.0676	-20.1617	-19.8637	-19.856	-19.8735
-4.20666	-5.02361	-4.83493	-4.83376	-5.38063	-4.96236	-4.71186	-5.19552	-5.37901	-4.86456	-4.685
0.771525	-0.22281	-0.07285	0.185544	-0.33092	0.273637	0.178447	-0.21928	-0.25914	0.270418	0.306662
10.20457	9.850208	10.17114	10.11359	9.910392	10.15309	10.10738	9.988073	9.972345	9.986968	10.05102
45.15433	44.64176	44.9434	44.909	44.9559	45.12949	45.00409	44.84692	44.99299	45.15949	45.06566
59.97074	59.51571	59.92597	60.15467	59.77095	60.11518	60.24357	59.63743	60.16791	60.0365	60.3525
83.2818	83.51892	86.54906	86.99451	84.67087	85.56649	86.04664	84.72545	85.79736	85.14746	87.14824

4. Simulation results

The proposed method is simulated using Matlab to examine its performance in DOA estimation. In this paper, it is assumed that the incoming signals are far field wideband signals are based on IEEE 802.15.4a standard [19] and they can be divided into 11 bins ($N_f = 11$), in which $f_L = 5.944\, GHz$ and $f_H = 10.234\, GHz$ are the minimum and maximum frequency of wideband signal spectrum, respectively. Moreover, they are also assumed as a sum of complex exponentials as follow

$$s(t) = a(t) \sum_{i=1}^{N_f} \exp\{j(2\pi f_i t + p_i)\} \qquad (19)$$

where the amplitude $a(t)$ is a Rayleigh random variable; the phase p_i is uniformly distributed in $[-\pi \div \pi]$ and N_f is the number of frequency components of wideband incoming signal.

In this research, a 6 - elements Nested antenna array ($D = 6$), in which $D_1 = 3$ and $D_2 = 3$, element spacing of inner array $d_1 = 0.5\lambda_{max}$, where $\lambda_{max} = \frac{c}{f_{max}}$ with f_{max} is the maximum frequency of all bins and the Pencil parameter is chosen $L = 9$. In order to evaluate the accuracy of the algorithm, the Root Mean Square Error (RMSE) is used. RMSE can be defined as

$$RMSE = \sqrt{\frac{\sum_{i=1}^{K} \left(x_i - x_i^{'}\right)^2}{K}} \qquad (20)$$

where x_i is the expected value and $x_i^{'}$ is the estimated value of measurement object i^{th} and K is the number of measurement objects. In our research, the measurement object is the DOA information.

In the first simulation, the proposed RDF system is executed to estimate the DOAs of eight incoming signals at $-50^o, -20^o, -5^o, 0^o, 10^o, 45^o, 60^o, 85^o$ in the AWGN channel with $SNR = 3dB$. The simulation results in each bin are presented in the table 1. This table shows that the DOAs are estimated accurately for all bins. And in order to get the best result, the average value in each row is calculated and chosen as the final estimated DOA information.

However, DOA estimation results are the numerical values as in Eq.18, therefore, in order to demonstrate visually the results, estimated DOA values will be illustrated in XOY plane, in which the X - Axis is the DOA of incoming signal and the Y - Axis is the indicating factor. This factor is set to 1 corresponding to the estimated DOA in X - Axis. The estimated result is shown in Fig.2. This figure indicates that all DOA of eight incoming signals have been successfully determined with very small error. Thus, the DF system is able to estimate a larger number of DOAs than number of antenna elements. Obviously, it is a considerable advantage of NAA in comparison with ordinary antenna array such as ULA and UCA where the number of DOA must be smaller than the number of antenna element. Moreover, it can be seen that DOA information can be extracted with only one snapshot. This is the most significant advantages of TFBMP that other methods such as MUSIC, ESPRIT… cannot do.

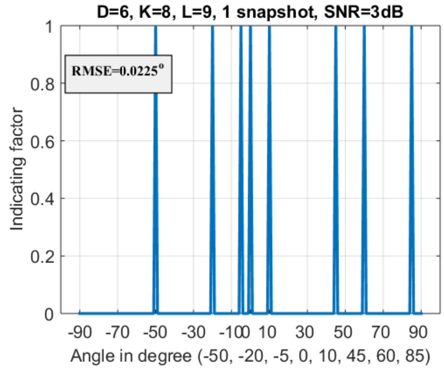

Fig. 2. DOA estimation result of eight incoming signals with only one snapshot

Simulation result shown in Fig.3 presents the influence of the number of snapshots on the performance of this algorithm. Clearly, when the number of snapshots are increased, the accuracy of this method increases. However, it can be seen that when the number of snapshot is more than 50, change in accuracy of the algorithm is trivial. Furthermore, the computation time will significantly increases when the number of snapshot increases. Therefore, it should be taken into account the trade-off between the computation time and the accuracy of the algorithm.

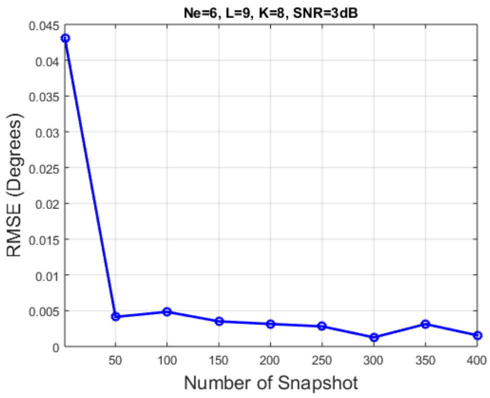

Fig. 3. Impact of number of snapshots on DOA estimation accuracy

Figure 4 depicts the performance of system in the AWGN channel with the variable SNRs from −10dB to 30dB with one snapshot. The RMSE is presented to prove the accuracy of system performance. It is quite good in white noise environment although with one snapshot.

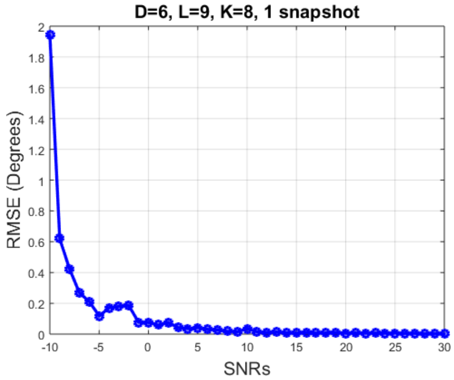

Fig. 4. DOA estimation accuracy in AWGN channel with varying SNR

5. Conclusions

In this paper, a novel RDF system for wideband signal using NAA and TFBMP method is proposed. The power of this system is that it can produce exactly DOA information of incoming wideband signals which are more than the number of antenna element with only one snapshot. By this way, the proposed system will reduce quantity of antenna element in comparison with other systems. Moreover, with one snapshot, the wideband RF signal can be directly converted to digital domain by highpass sampling. Therefore, the proposed system can be implemented for real time all digital RDF system.

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