Dynamic Analysis of Complex Composite Tubes by Continuous Element Method

Le Thi Bich Nam[1, *]*, Nguyen Manh Cuong*[1]*, Tran Ich Thinh*[1]*,*
Duong Pham Tuong Minh[2]*, Le Quang Vinh*[3]

[1] *School of Mechanical Engineering, Hanoi University of Science and Technology*
[2] *Faculty of Mechanical Engineering, Thai Nguyen University of Technology*
[3] *Department of Mechanical Engineering, Viet tri University of Industry*

Received: December 08, 2016; accepted: June 9, 2017

Abtract

This paper presents a new continuous element model for studying the dynamic behavior of complex tubes composed by cylindrical shell, annular plate and conical shell which are widely used in automobile exhausts, sewer pipes and pipelines. Based on the analytical solutions of the system of differential equations for cylinders, cones and annular plates, the dynamic stiffness matrix for complex composite tubes has been established and assembled. Natural frequencies and harmonic responses for composite tubes have been calculated and validated by comparing with the literature and with Finite Element Method. The assembly procedure of Continuous Element Method demonstrated remarkable advantages in terms of precision, volume of data storage, calculating time and larger range of studied frequencies.

Keywords: Complex shell, tube, Continuous Element Method, composite shell

1. Introduction

The complex composite tubes have many applications in various branches of engineering such as automobile, aeronautical, marine, civil and power industry. Hence, the comprehension of dynamic behaviors of such structures is of great important in order to design and fabric safer and stronger composite shell structures. Different methods for analyzing free vibrations of isotropic and composite joined conical-cyclindrical shells have been applied. Liang et al. [1] have investigated the dynamic characteristics of a symmetric cross-ply laminated conical shell with an annular plate at the top end using the transfer matrix method. The free vibration of joined isotropic conical-cyclindrical shells has been solved by Irie et al. [2] using the transfer matrix approach. Caresta and Kessissoglou [3] have analyzed the free vibrations of joined truncated conical-cyclindrical shells using a power series solution and a wave solution method. Kouchakazadeh and Shakouri [4] presented a study dealing with vibrational behavior of two joined cross-ply laminated conical shells, joined cylindrical-conical shells using thin-walled shallow shell in which the expressions among stress resultants and deformations are extracted as continuity condition at the joining section of the cones. Recently, Qu et al. [5] proposed free vibration characteristics of conical–cylindrical–spherical shell combinations with ring stiffeners by using a modified variational method, Reissner–Naghdi's thin shell theory in conjunction with a multilevel partition technique. Kang [6] investigated the free vibration of joined thick conical–cylindrical shells with variable thickness using a three-dimensional Ritz method.

The main draw-back of traditional methods like FEM is the discretization of the domain which causes errors in dynamic analysis, especially in medium and high frequencies. The Continuous Element Method (CEM) or Dynamic Stiffness Method (DSM) based on the closed form solution of the system of differential equations of the structure is developed to overcome these difficulties. The CE models for composite cylindrical shell presented in works of Tran Ich Thinh and Nguyen Manh Cuong [7] imposes a considerable advancement of the study on CEM for composite structures. Recently, the new research for thick laminated composite joined cyclindrical-conical shells by Nguyen Manh Cuong et al. [8] has emphasized the strong capacity of DSM in assemblying complex structure. Le Thi Bich Nam et al. [9] have proposed a new continuous element formulation for cross-ply annular plates.

However, there is a lack of researches on combined cylinder-annular plate-cone structures. The purpose of this paper is presenting a new continuous element for thick combined cross-ply laminated tubes composed by cylinder, annular plate and conical shells taking into account the effect of shear deformations and rotational inertia as well as the continuity conditions at joined sections. Obtained results using this model have been compared with

[*] Corresponding author: Tel.: (=84) 24.3868.0103
Email: nam.lethibich@hust.edu.vn

those of FEM and with available results in other investigations and good agreement was obtained and advantages of CE model have been confirmed.

2. Formulation of thick cross-ply composite shells of revolution

2.1. Theory of composite conical shells

2.1.1. Constitutive relations

The plane stress-reduced stiffnesses of a laminate composite composed by N orthotropic layers are:

$$Q_{11}=\frac{E_1}{1-\nu_{12}\nu_{21}},\ Q_{12}=\frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}},\ Q_{22}=\frac{E_2}{1-\nu_{12}\nu_{21}},$$
$$Q_{66}=G_{12},\ Q_{44}=G_{23},\ Q_{55}=G_{13} \qquad (1)$$
$$A_{ij}=\sum_{k=1}^{N}\bar{Q}_{ij}^{k}(z_{k+1}-z_k),\ A_{ij}=\frac{1}{4}\sum_{k=1}^{N}\bar{Q}_{ij}^{k}(z_{k+1}^{4}-z_k^{4})\ (i,j=4,5),$$
$$B_{ij}=\frac{1}{2}\sum_{k=1}^{N}\bar{Q}_{ij}^{k}(z_{k+1}^{2}-z_k^{2}),\ D_{ij}=\frac{1}{3}\sum_{k=1}^{N}\bar{Q}_{ij}^{k}(z_{k+1}^{3}-z_k^{3})\ (i,j=1,2,6)$$

where $E_i, G_{ij}, \upsilon_{12}, \upsilon_{21}$: elastic constants of the k^{th} layer, A_{ij}, B_{ij}, D_{ij}: laminate stiffness coefficients and z_{k-1} and z_k are the boundaries of the k^{th} layer.

2.1.2. Strains, stress and forces resultant

Following the First Order Shear Deformation Therory (FSDT), the displacement components are written as (see Fig. 1):

$$u(s,\theta,z,t)=u_0(s,\theta,t)+z\varphi_s(s,\theta,t),$$
$$v(s,\theta,z,t)=v_0(s,\theta,t)+z\varphi_\theta(s,\theta,t)$$
$$w(s,\theta,z,t)=w_0(s,\theta,t) \qquad (2)$$

with u_0, v_0, w_0: displacements of the point M_o at the median radius of the shell and φ_S, φ_θ : rotations of a transverse normal about the θ -axis and s-axis.

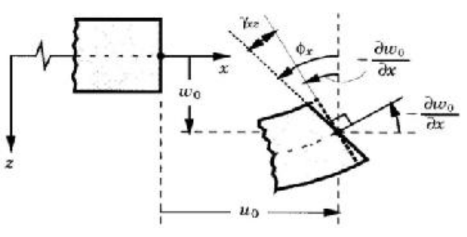

Fig. 1. Geometries of a shell edge by FSDT

2.1.2. The strain-displacement relations of conical shell

$$\varepsilon_S=\frac{\partial u_0}{\partial s} \qquad k_{S\theta}=\frac{1}{R}\frac{\partial\varphi_S}{\partial\theta}+\frac{\partial\varphi_\theta}{\partial s}-\frac{\sin\alpha}{R}\varphi_\theta \qquad k_S=\frac{\partial\varphi_S}{\partial s}$$
$$\varepsilon_\theta=\frac{1}{R}\left(u_0\sin\alpha+\frac{\partial v_0}{\partial\theta}+w_0\cos\alpha\right),\ \gamma_{\theta Z}=\frac{-\cos\alpha}{R}v_0+\frac{1}{R}\frac{\partial w_0}{\partial\theta}+\varphi_\theta$$
$$k_\theta=\frac{1}{R}\left(\varphi_S\sin\alpha+\frac{\partial\varphi_\theta}{\partial\theta}\right) \qquad \varepsilon_{S\theta}=\frac{\partial v_0}{\partial s}+\frac{1}{R}\frac{\partial u_0}{\partial\theta}-\frac{\sin\alpha}{R}v_0,$$

with $R=R_1+s.\sin\alpha$ (3)

2.1.3. Force resultants–displacement relationships

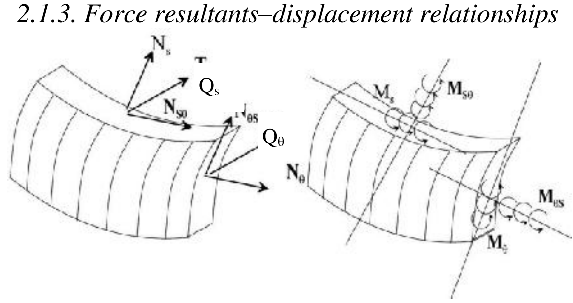

Fig. 2. Force and moments resultants on shells

Fig. 2 illustrates the force and moment resultants on a shell. The forces-displacements expressions for laminated composite conical shell are written as follows:

$$N_S=A_{11}\frac{\partial u_0}{\partial s}+\frac{A_{12}}{R}\left(u_0\sin\alpha+\frac{\partial v_0}{\partial\theta}+w_0\cos\alpha\right)+B_{11}\frac{\partial\varphi_S}{\partial s}+\frac{B_{12}}{R}\left(\varphi_S\sin\alpha+\frac{\partial\varphi_\theta}{\partial\theta}\right)$$
$$Q_S=kA_{55}\left(\frac{\partial w_0}{\partial s}+\varphi_S\right),\ Q_\theta=kA_{44}\left(\frac{-\cos\alpha}{R}v_0+\frac{1}{R}\frac{\partial w_0}{\partial\theta}+\varphi_\theta\right) \qquad (4)$$
$$N_\theta=A_{12}\frac{\partial u_0}{\partial s}+\frac{A_{22}}{R}\left(u_0\sin\alpha+\frac{\partial v_0}{\partial\theta}+w_0\cos\alpha\right)+B_{12}\frac{\partial\varphi_S}{\partial s}+\frac{B_{22}}{R}\left(\varphi_S\sin\alpha+\frac{\partial\varphi_\theta}{\partial\theta}\right)$$
$$N_{S\theta}=A_{66}\left(\frac{\partial v_0}{\partial s}+\frac{1}{R}\frac{\partial u_0}{\partial\theta}-\frac{\sin\alpha}{R}v_0\right)+B_{66}\left(\frac{1}{R}\frac{\partial\varphi_S}{\partial\theta}+\frac{\partial\varphi_\theta}{\partial s}-\frac{\sin\alpha}{R}\varphi_\theta\right)$$
$$M_S=B_{11}\frac{\partial u_0}{\partial s}+\frac{B_{12}}{R}\left(u_0\sin\alpha+\frac{\partial v_0}{\partial\theta}+\frac{w_0\cos\alpha}{R}\right)+D_{11}\frac{\partial\varphi_S}{\partial s}+\frac{D_{12}}{R}\left(\varphi_S\sin\alpha+\frac{\partial\varphi_\theta}{\partial\theta}\right)$$
$$M_\theta=B_{12}\frac{\partial u_0}{\partial s}+\frac{B_{22}}{R}\left(u_0\sin\alpha+\frac{\partial v_0}{\partial\theta}+w_0\cos\alpha\right)+D_{12}\frac{\partial\varphi_S}{\partial s}+\frac{D_{22}}{R}\left(\varphi_S\sin\alpha+\frac{\partial\varphi_\theta}{\partial\theta}\right)$$
$$M_{S\theta}=B_{66}\left(\frac{\partial v_0}{\partial s}+\frac{\partial u_0}{R\partial\theta}-\frac{\sin\alpha}{R}v_0\right)+D_{66}\left(\frac{1}{R}\frac{\partial\varphi_S}{\partial\theta}+\frac{\partial\varphi_\theta}{\partial s}-\frac{\sin\alpha}{R}\varphi_\theta\right)$$

where k is the shear correction factor (k=5/6), (N_s, N_θ, $N_{s\theta}$) are force resultants, (M_s, M_θ, $M_{s\theta}$) are moment resultants and (Q_s,Q_θ) are shear force resultants at s and θ directions, respectively

2.1.4. Equations of motion

The equations of motion using the FSDT for laminated composite conical shell are [8]:

$$\frac{\partial N_S}{\partial s}+\frac{\sin\alpha}{R}(N_S-N_\theta)+\frac{1}{R}\frac{\partial N_{S\theta}}{\partial\theta}=I_0\ddot{u}_0+I_1\ddot{\varphi}_S$$
$$\frac{\partial N_{S\theta}}{\partial s}+\frac{2\sin\alpha}{R}N_{S\theta}+\frac{1}{R}\frac{\partial N_\theta}{\partial\theta}+\frac{\cos\alpha}{R}Q_\theta=I_0\ddot{v}_0+I_1\ddot{\varphi}_\theta$$
$$\frac{\partial M_S}{\partial s}+\frac{\sin\alpha}{R}(M_S-M_\theta)+\frac{1}{R}\frac{\partial M_{S\theta}}{\partial\theta}-Q_S=I_1\ddot{u}_0+I_2\ddot{\varphi}_S \qquad (5)$$
$$\frac{\partial M_{S\theta}}{\partial s}+\frac{2\sin\alpha}{R}M_{S\theta}+\frac{1}{R}\frac{\partial M_\theta}{\partial\theta}-Q_\theta=I_1\ddot{v}_0+I_2\ddot{\varphi}_\theta$$
$$\frac{\partial Q_S}{\partial s}+\frac{1}{R}\frac{\partial Q_\theta}{\partial\theta}+\frac{\sin\alpha}{R}Q_S-\frac{\cos\alpha}{R}N_\theta=I_0\ddot{w}_0$$

with $I_i=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\rho^{(k)}z^i dz\quad (i=0,1,2)$

where $\rho^{(k)}$: material mass density of the k^{th} layer.

The equations for cylindrical shells and annular plates can be derived from equations (4), (5) by using $\alpha = 0^o$ and $\alpha = 90^o$ respectively.

3. Continuous element for thick composite shells of revolution

3.1. State vector

The state-vector for the investigated conical shells is $\mathbf{y}^T = \{u_0, v_0, w_0, \varphi_S, \varphi_\vartheta, N_S, N_{S\vartheta}, Q_S, M_S, M_{S\vartheta}\}^T$. Using the Fourier series expansion, state variables for the circumferential wave *m* are written as:

$$\{u_o(s,\vartheta,t), w_o(s,\vartheta,t), \varphi_s(s,\vartheta,t), N_S(s,\vartheta,t), Q_S(s,\vartheta,t), M_S(s,\vartheta,t)\}^T = \sum_{m=1}^{\infty}\{u_m(s), w_m(s), \varphi_{sm}(s), N_{S_m}(s), Q_{Sm}(s), M_{S_m}(s)\}^T \cos m\vartheta\, e^{i\omega t}$$

$$\{v_o(s,\vartheta,t), \varphi_\vartheta(s,\vartheta,t), N_\vartheta(s,\vartheta,t), Q_\vartheta(s,\vartheta,t), M_\vartheta(s,\vartheta,t)\}^T = \sum_{m=1}^{\infty}\{v_m(s), \varphi_{\vartheta m}(s), N_{\vartheta m}(s), Q_{\vartheta m}(s), M_{\vartheta m}(s)\}^T \sin m\vartheta\, e^{i\omega t} \quad (6)$$

Substituting (6) in (4) and (5), a system of differential equations in the *s*-coordinate can be expressed in the matrix form as: $dy_m/ds = \mathrm{A}_m(s,\omega)\mathrm{y}_m$

with $\mathbf{A}_m$ is a 10x10 matrix (7)

3.2. Dynamic stiffness matrix K(ω)

Then, the dynamic stiffness matrix $\mathbf{K}(\omega)_m$ for conical shell is determined by [8], [9]:

$$\mathrm{K}(\omega)_m = \begin{bmatrix} \mathrm{T}_{12}^{-1}\mathrm{T}_{11} & -\mathrm{T}_{12}^{-1} \\ \mathrm{T}_{21} - \mathrm{T}_{22}\mathrm{T}_{12}^{-1}\mathrm{T}_{11} & \mathrm{T}_{22}\mathrm{T}_{12}^{-1} \end{bmatrix}_m$$

$$\text{with } T_m = e^{\int_0^L A(s,\omega)ds} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \quad (8)$$

4. Continuous element method for composite tubes

Consider a joined cylindrical shell-annular plate- conical shell in Fig. 3 with R_1: radius of the cylinder, R_2 and R_3: small and large radius of the cone. L_1 and L_2: lengths of the cylinder and cone along its generator, h: thickness of shells, α: half cone angle.

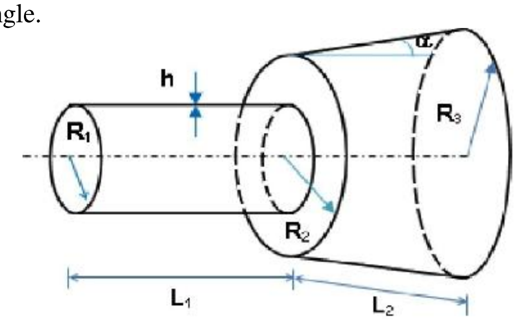

Fig. 3. Geometry of a tube composed by cylinder, annular plate and cone

4.1. Continuity conditions

The continuity conditions for assembling tubes composed by cylinder, cone and annular plate are expressed as follows [3]:

$$u_1 = u_2 \cos\alpha - w_2 \sin\alpha, \quad v_1 = v_2,\ \varphi_{s1} = \varphi_{s2}, \quad (9)$$
$$w_1 = u_2 \sin\alpha + w_2 \cos\alpha,\ Q_{s1} = N_{s2}\sin\alpha + Q_{s2}\cos\alpha,$$
$$N_{s1} = N_{s2}\cos\alpha - Q_{s2}\sin\alpha, \quad M_{s1} = M_{s2},\ M_{s\vartheta 1} = M_{s\vartheta 2}$$

4.2. Assembly procedure

The performant assembly procedure of Continuous Elements presented in our previous research [8] is used to construct the Dynamic Stiffness Matrix for combined cylindrical-annular plate-conical shells. First, the dynamic stiffness matrix for cylinder, annular plate and cone must be evaluated separately. Then the dynamic stiffness matrix for the tube can be constructed by employing the assembly procedure similar to those of FEM for assembling stiffness matrix illustrated in Fig. 3.

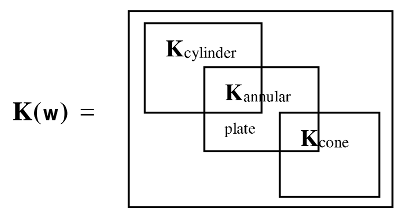

Fig. 2. Construction of dynamic stiffness matrix for composite tubes

5. Results and discussions

5.1. Modal analysis

A Matlab program based on presented formulations is developed for analyzing the vibration of a joined cylinder-annular plate-cone tube. First, a composite conical shell with an annular plate at the top end subjected to the free-clamped boundary condition is taken for example. Geometric properties are R = 200 mm, R_1 = 180 mm, R_2 = 100 mm, h = 2 mm, and L_2 = 100 mm where R and R_1: radius of large and small edges of the cone, R_2: inner radius of the annular plate, material parameters are E_1 = 135 GPa, E_2 = 8.8 GPa, $G_{12} = G_{13} = G_{23}$ = 4.47 GPa and υ_{12} = 0.33, ρ = 1600 kg/m^3 with layer configuration [90/0/0/90] (Material 1). Obtained dimensionless frequencies $\Omega = R(\rho h\omega^2/D_{11})^{1/4}$ calculated by the present formulation and by Liang et al. [1] using both FEM and Transfer matrix method are compared in Table 1. Here m represents the number of circumferential waves and n the number of axial half waves. It is easy to remark that the present method gives high precise results which are very closed to those from Liang et al. [1], especially to the transfer matrix method solution. Tiny differences (0.07% to 3.49%) between our formulation and those in [1] confirm that the present solution is exact and the present formulations can be applied to study the dynamic behavior of tubes.

There is no comparative data from literature for complex composite tubes. Therefore, obtained natural frequencies of our CE program will be validated with respect to those of FEM. For FE models, the Ansys SHELL181 elements with the 180x20 mesh assure good results after a convergence test. The parameters of the considered tube are: h = 0.002m, R_1 = 0.1m, R_2 = 0.18m, R_3 = 0.2 m, $L_1 = L_2 \cos\alpha$ = 0.1 m, Material 1. The comparison of CE and FE results for clamped-clamped (C-C), free-clamped (F-C) and supported-clamped (S-C) composite tubes for various vibration modes is illustrated in Table 2. It is seen from this table that good agreements are noticed between CE and FE solutions. CE model give excellent solutions which are close to FE results with small errors varying from 0% to 3.75%. The precision of our model is valid for analyzing the considered tube with different modes and boundary conditions.

5.2. Harmonic responses

This section confirms important advantages of CE models with respect to other methods for complex structures. Fig. 3 shows the comparison of FE and CE harmonic responses for the studied tube subjected to the C-C (left) and F-C boundary condition (right).

It is seen that using a raw mesh (10x80) the discrepancies between FE and CE curves are noticed from 1495.5 Hz for the 1st test and at 1660 Hz for the second one. More precise FE solutions are obtained by using a finer meshing (20x120) because the FE curves coincide with CE one within a larger frequency range. However, important differences between two solutions still occur from 2556 Hz for the 1st test and from 2413,5 Hz for the second one. It is clear to remark that FE models converge towards those of CE when reducing the element sizes.

Table 1. Comparison of $\Omega = R\sqrt[4]{\rho h\omega^2 / D_{11}}$ for a cross-ply combined conical shell-annular plate with layer scheme $[90^o/0^o]_s$, Material 1 and R_1 = 200mm, $h_1 = h_2$ = 2 mm, R_2 = 180mm, $\alpha_1 = -11.77^o$

	Method	m=1	Errors (%)	m=2	Errors (%)
n=0	FEM [1]	5.4672	0.18	11.745	0.35
	Transfer matrix [1]	5.4575	0	11.786	0
	CEM	5.3911	1.23	11.7452	3.49
n=1	FEM [1]	5.9070	0.88	11.876	0.65
	Transfer matrix [1]	5.8554	0	11.799	0
	CEM	5.8660	0.18	11.7904	0.07
n=2	FEM [1]	7.1143	0.79	12.048	0.39
	Transfer matrix [1]	7.0588	0	12.001	0
	CEM	7.1306	1.00	12.117	0.95

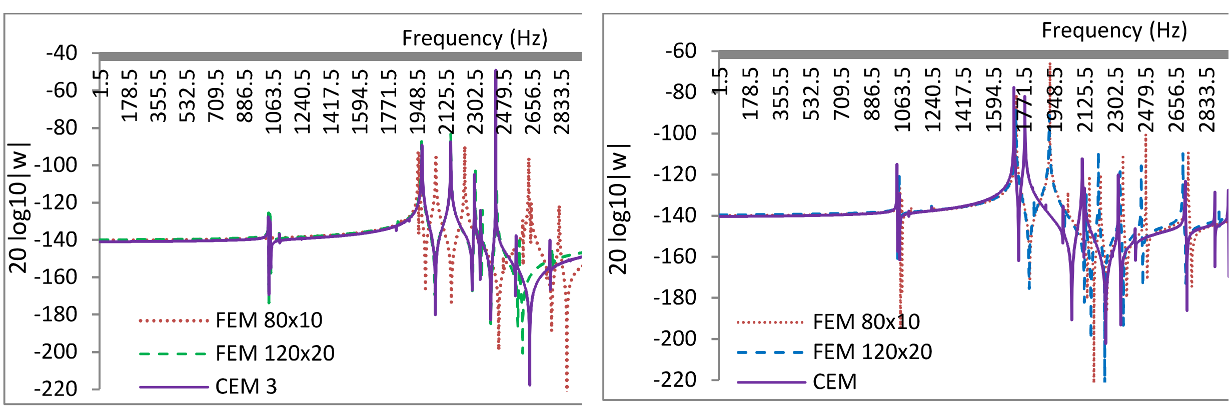

Fig. 3. Comparison of harmonic responses computed by FEM and by CEM for a C-C cylinder-annular plate-cone tube (Left) and a F-C cylinder-annular plate-cone tube (Right)

Table 2. Comparison of natural frequencies for a composite tube with C-C, F-C and S-C boundary conditions (h = 0.002m, R_1 = 0.1m, R_2 = 0.18m, R_3 = 0.2 m, $L_1 = L_2 \cos\alpha$ = 0.1 m. Material 1)

Boundary Condition	Mode		Frequency (Hz)		
	n	m	FEM (180x20)	CEM	Error (%)
C-C	1	1	1036.1	1035.7	0.04
	1	2	1047.4	1047.7	0.03
	1	3	1102.6	1104.2	0.14
	1	4	1235.6	1238.2	0.21
	1	5	1463.2	1464.9	0.12
	1	6	1665.8	1660.2	0.34
	1	7	1755.2	1745.7	0.54
	2	5	1825.5	1820.7	0.26
F-C	1	1	184.51	187.8	1.75
	1	2	268.54	279.0	3.75
	1	3	619.83	626.3	1.03
	2	2	1063.5	1065.3	0.17
	2	1	1072.2	1072.2	0.00
	2	0	1120.4	1121.5	0.10
	2	3	1122.2	1124.3	0.19
	1	4	1144.7	1147.9	0.28
S-C	1	1	1035.9	1035.5	0.04
	1	0	1043.0	1042.6	0.04
	1	2	1047.2	1048.9	0.16
	1	3	1101.7	1105.9	0.38
	1	4	1236.1	1241.8	0.46
	1	5	1466.8	1470.1	0.22
	1	6	1674.7	1662.1	0.76
	2	1	1725.4	1714.6	0.63

Using a minimum meshing, CE model demonstrates considerable advantages when dealing with complex structures in terms of calculating time and the saving of data storage volume. For example, the required time for plotting FE harmonic response curves of the 1st test are 143 minutes (10x80 FE mesh), 289 minutes (20x120 FE mesh) but is only 81 minutes for CE model with only 3 elements. These CE advantages are verified for different boundary conditions and for all frequency range.

6. Conclusions

This research has presented a new Dynamic Stiffness Matrix for thick combined cross-ply laminated tube composed by cylinder, annular plate and conical shells taking into account the effect of shear deformations and rotational inertia as well as the continuity conditions at joined sections.

The exactness and validity of continuous model for different properties of geometry, vibration mode and boundary conditions have been confirmed by the excellent agreements between obtained results with those published by other researches and by Finite Element Method. In conclusion, the proposed Continuous Element model consists an interesting approach to calculate the natural frequencies of thick combined cross-ply laminated tube with high accuracy, especially for medium and high frequencies where other current methods give unreliable solutions. With a minimum meshing for complex structures, our model accelerates the calculating speed and saves the volume of data storage.

The introduced Continuous Element can be developed to resolve the problem of shells on elastic foundation and containing flowing fluid, shells on non-homogenous elastic foundations, shells with damping or shells with stiffeners.

References

[1] Sen Liang, H.L. Chen, Tianning Chen, Michael Yu Wang (2007). The natural vibration of a symmetric cross-ply laminated composite conical-plate shell, Comp. Struct. 80 (2007) 265–278.

[2] T. Irie, G. Yamada and Y. Muramoto (1984). Free vibration of joined conical-cylindrical shells, Journal of Sound and Vibration (1984) 95(l), 31-39.

[3] Mauro Caresta, Nicole J. Kessissoglou (2010). Free vibrational characteristics of isotropic coupled cylindrical–conical shells, Journal of Sound and Vibration 329 (2010) 733–751.

[4] M.A.Kouchakzadeh, M.Shakouri (2014). Free vibration analysis of joined cross-ply laminated conical shells, International Journal of Mechanical Sciences 78(2014)118–125.

[5] Yegao Qu, Shihao Wu, Yong Chen, Hongxing Hua (2013). Vibration analysis of ring-stiffened conical–cylindrical–spherical shells based on a modified variational approach, International Journal of Mechanical Sciences 69 (2013) 72–84.

[6] Jae-Hoon Kang (2012). Three-dimensional vibration analysis of joined thick conical-cylindrical shells of revolution with variable thickness, J. of Sound and Vibration 331(2012) 4187–4198.

[7] Tran Ich Thinh, Nguyen Manh Cuong (2013). Dynamic stiffness matrix of continuous element for vibration of thick cross-ply laminated composite cylindrical shells. Compos Struct; 98:93–102.

[8] Nguyen Manh Cuong, Tran Ich Thinh and Vu Quoc Hien (2014). Vibration analysis of cross-ply composite joined conical-cylindrical shells by Continuous Element Method, Proceedings of the International Conference on Engineering Mechanics and Automation-ICEMA3, Pages 401-408.

[9] Le Thi Bich Nam, Nguyen Manh Cuong, Tran Ich Thinh (2014). Continuous Element formulation for vibration of thick composite annular plates and rings, Tuyển tập công trình Hội nghị Cơ học kỹ thuật toàn quốc Kỷ niệm 35 năm thành lập Viện Cơ học, 10/4/1979-10/4/2014, (2), Pages 319-324