Bi-Level Optimization Model for Calculation of LMP Intervals Considering the Joint Uncertainty of Wind Power and Demand

Pham Nang Van
Hanoi University of Science and Technology – No. 1, Dai Co Viet Str., Hai Ba Trung, Ha Noi, Viet Nam
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Abstract

In the electricity market operation, electricity prices or Locational Marginal Prices (LMP) vary according to both electric demand and the penetration level of the wind power. The variable domain identification of LMP plays a very important role for market participants to assess and mitigate the risk on account of the combined uncertainty of wind power and demand. Traditionally, the Monte Carlo simulation (MCS) method can be used in order to determine the variable intervals of LMP. However, in this paper, author deploys a bi-level optimization model to calculate the upper and lower bounds of LMP when considering the combined uncertainty of wind power generation and demand. The objective function of the upper-level optimization problem is to maximize (or minimize) LMP at a node whereas the objective function of the lower-level optimization problems is to calculate the optimal power generation of the units participating in supplying the load.

Key words**:** electricity market, mathematical program with equilibrium constraints (MPEC), mixed-integer linear programming (MILP), joint uncertainty of wind power and demand, Locational Marginal Prices (LMP).

1. Introduction

Currently, many countries around the world, including Vietnam, have been operating wholesale electricity markets. In the wholesale electricity market, the market participants are generation companies (GENCOS) and distribution companies (DISCOS). The market operator collects generating offers by producers, load bids by consumers and clears the market by maximizing the social welfare [1]-[2].

The uncertainty from wind output has brought unprecedented challenges to the optimal operation of the electricity market. The power system operation has been dealing with the uncertainty of load; however, wind output is characterized with large uncertainties and low prediction precision [3]. On the other hand, load demand has an intrinsic pattern and thus the load prediction, especially, in short-term, has a significantly high forecast accuracy [3]. Therefore, the optimal operation and dispatching model considering stochastic wind power output has been a hot topic for research.

Reference [4] studied the effect of wind integration and wind uncertainty on power system reliability, using an ARMA model to analyze short-term wind forecast. Reference [5] studied the impact of stochastic wind power on the unit commitment (UC) problem and constructed a UC stochastic optimization problem with the objective to minimize the expected operation cost. In reference [6], the influence of distributed generation on a heavily loaded distribution system with a wind forecast model based on statistics is tackled. A mixed-integer stochastic optimization model is established in [7] where the wind uncertainty is modeled with ARMA as well as Latin hypercube sampling and a scenario reduction method is adopted to simplify the computation.

The first step to investigate the effect of uncertainty is to model the uncertain wind output by using a variety of methods, for instance, probability distribution model [8], fuzzy model [9] and interval number model [10]. In the next steps, different optimization models are applied to find the solution.

To make payments in the electricity market, locational marginal price (LMP) are calculated. The difference in LMPs between two nodes of a branch depends upon the congestion and losses on that branch [2]. The locational marginal pricing methodology is widely used in electricity markets to determine the electricity prices and to evaluate the transmission congestion cost [11]-[12]. Step change characterizes of LMP under system load variation has been identified and discussed [13]. Moreover, the concept of critical load level (CLL) is defined and employed for load frequency control [13]. Based on a similar idea, the investigation of the impact of variable wind power

* Corresponding author: Tel: (+84) 988266541
Email: van.phamnang@hust.edu.vn

outputs on LMPs must be worth launching. It is important to find a method to efficiently obtain the wholesale electricity price intervals under the variation of both wind power output and demand.

This paper proposes an approach to determine the intervals of LMP using a bi-level optimization model, which is similar to the interval number-based optimization model regarded as the optimization of optimization.

The next sections of the article are organized as follows. In section 2, the authors present bi-level optimization model to determine LMP intervals. In addition, the authors also describe the solution to solve this bi-level optimization problem including the procedure of transferring it into a Mathematical Program with Equilibrium Constraints (MPEC) problem and the conversion from MPEC to a Mixed-Integer Linear Programming (MILP). Section 3 demonstrates the simulation results and numerical analyses of PJM 5-bus system and IEEE 24-bus system. Some conclusions are given in section 4.

2. LMP intervals under the joined uncertainty of wind power and demand

2.1 Scenario-based market clearing model to integrate wind power

Economic Dispatch (ED) in electricity market is carried out by Independent System Operators (ISOs) to clear market as well as determine LMPs and output of generating units. In this paper, the DCOPF-based approach without losses is employed to model the electricity market and estimate LMPs. This DCOPF including wind power for one scenario is a linear programming (LP) problem presented as follows:

$$\min \sum_{i=1}^{N}\left(c_{Gi}P_{Gi}^{s}+c_{Wi}P_{Wi}^{s}\right) \tag{1}$$

$$s.t. \sum_{i=1}^{N}\left(P_{Gi}^{s}+P_{Wi}^{s}\right)=\sum_{i=1}^{N}P_{Di} : \lambda^{s} \tag{2}$$

$$\begin{gathered}-Limit_l \le \sum_{i=1}^{N} GSF_{l-i}\left(P_{Gi}^{s}+P_{Wi}^{s}-P_{Di}\right)\le Limit_l : \\ \mu_l^{s,\min},\mu_l^{s,\max},\forall l=\overline{1,M}\end{gathered} \tag{3}$$

$$P_{Gi}^{\min}\le P_{Gi}^{s}\le P_{Gi}^{\max} : \omega_i^{s,\min},\omega_i^{s,\max},\forall i=\overline{1,N} \tag{4}$$

$$0\le P_{Wi}^{s}\le P_{Wi}^{s,\max} : \varphi_i^{s,\min},\varphi_i^{s,\max},\forall i=\overline{1,N} \tag{5}$$

where N is the number of buses; M is the number of lines; c_{Gi} and c_{Wi} are energy prices offered by conventional generation and wind power, respectively; P_{Gi}^{s} and P_{Wi}^{s} are power outputs of the conventional generating unit and wind power, respectively; P_{Di} are the consumed power of demand i; GSF is the generation shift factor matrix; $P_{Gi}^{\min}$ and $P_{Gi}^{\max}$ are the upper and lower bounds of the convention generation output; $P_{Wi}^{s,\max}$ is the maximum available wind power output and the variables on the right side of the colon are the dual variables associated with the equality and inequality constraints on the left.

The LMP at bus i for one scenario can be calculated from the Lagrange function of the above ED problem. This function and LMP are given by

$$\begin{aligned}\psi^{s} &= \sum_{i=1}^{N}\left(c_{Gi}P_{Gi}^{s}+c_{Wi}P_{Wi}^{s}\right)\\ &-\lambda^{s}\sum_{i=1}^{N}\left(P_{Gi}^{s}+P_{Wi}^{s}-P_{Di}\right)\\ &-\sum_{l=1}^{M}\mu_l^{s,\min}\left(\sum_{i=1}^{N}GSF_{l-i}\left(P_{Gi}^{s}+P_{Wi}^{s}-P_{Di}\right)+Limit_l\right)\\ &-\sum_{l=1}^{M}\mu_l^{s,\max}\left(Limit_l-\sum_{i=1}^{N}GSF_{l-i}\left(P_{Gi}^{s}+P_{Wi}^{s}-P_{Di}\right)\right)\\ &-\sum_{i=1}^{N}\omega_i^{s,\min}\left(P_{Gi}^{s}-P_{Gi}^{\min}\right)-\sum_{i=1}^{N}\omega_i^{s,\max}\left(P_{Gi}^{\max}-P_{Gi}^{s}\right)\\ &-\sum_{i=1}^{N}\varphi_i^{s,\min}P_{Wi}^{s}-\sum_{i=1}^{N}\varphi_i^{s,\max}\left(P_{Wi}^{s,\max}-P_{Wi}^{s}\right)\end{aligned} \tag{6}$$

$$LMP_i^{s}=\lambda^{s}+\sum_{l=1}^{M}GSF_{l-1}\left(\mu_l^{s,\min}-\mu_l^{s,\max}\right) \tag{7}$$

2.2 Bi-level optimization for determination LMP interval

Traditionally, the intervals of LMP are usually evaluated using Monte Carlo Simulation (MCS) approach. However, this approach requires a huge amount of computation time in comparison with the bi-level optimization approach in term of the same level of accuracy. The problem for calculation LMP intervals simultaneously considering the uncertainty of wind power generation and demand is an optimization problem constrained by a number of interrelated optimization problems depicted in Figure 1.

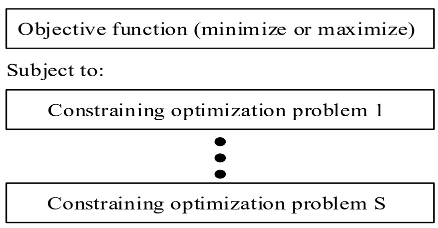

Fig. 1. Optimization problem constrained by a number of interrelated optimization problems

This bi-level optimization problem is formulated as follows:

$$Upper\ Level : \max \text{ (or min) } \sum_{s=1}^{S} p_s LMP_i^s \quad (8)$$

s.t.

$$\begin{aligned} &Lower\ level : Scenario-based\ ED\ opt.\ \text{model} \\ &\{(1)-(5)\}\ \forall s = \overline{1,S} \end{aligned} \quad (9)$$

$$P_{Di}^{\min} \le P_{Di} \le P_{Di}^{\max} \quad (10)$$

where $P_{Di}^{\min}$ and $P_{Di}^{\max}$ is the forecast upper and lower bounds of the consumed load, S is the number of scenarios, p_s is probability of scenario s.

2.3 Formulation as a MPEC

Given that the lower level optimization models are LP problems, the bi-level can be transformed into an MPEC by recasting the lower level problems as their Karush-Kuhn-Tucker (KKT) optimality conditions, which are added into the upper level problem as the additional complementarity constraints [15]. Figure 2 illustrates the structure of an MPEC considering KKT conditions as constraints.

This MPEC problem can be expressed as following:

$$\text{Objective function } (8) \quad (11)$$

s.t.

$$\text{Constraints in (2) and (10)} \quad (12)$$

$$c_{Gi} = \lambda^s + \sum_{l=1}^{M} GSF_{l-i}\left(\mu_l^{s,\min} - \mu_l^{s,\max}\right) + \omega_i^{s,\min} - \omega_i^{s,\max} \quad (13)$$

$$c_{Wi} = \lambda^s + \sum_{l=1}^{M} GSF_{l-i}\left(\mu_l^{s,\min} - \mu_l^{s,\max}\right) + \varphi_i^{s,\min} - \varphi_i^{s,\max} \quad (14)$$

$$0 \le \mu_l^{s,\min} \perp Limit_l + \sum_{i=1}^{N} GSF_{l-i}\left(P_{Gi}^s + P_{Wi}^s - P_{Di}\right) \ge 0 \quad (15)$$

$$0 \le \mu_l^{s,\max} \perp Limit_l - \sum_{i=1}^{N} GSF_{l-i}\left(P_{Gi}^s + P_{Wi}^s - P_{Di}\right) \ge 0 \quad (16)$$

$$0 \le \omega_i^{s,\min} \perp P_{Gi}^s - P_{Gi}^{\min} \ge 0 \quad (17)$$

$$0 \le \omega_i^{s,\max} \perp P_{Gi}^{\max} - P_{Gi}^s \ge 0 \quad (18)$$

$$0 \le \varphi_i^{s,\min} \perp P_{Wi}^s \ge 0 \quad (19)$$

$$0 \le \varphi_i^{s,\max} \perp P_{Wi}^{s,\max} - P_{Wi}^s \ge 0 \quad (20)$$

The MPEC optimization problem (11) – (20) can be converted to a MILP problem, which is conducted as in subsection 2.4.

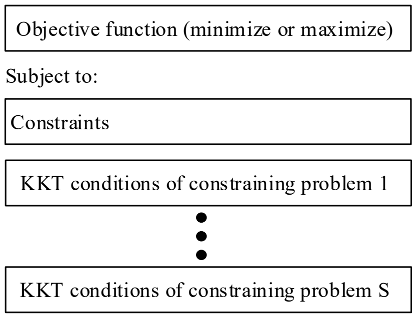

Fig. 2. Optimization problem constrained by sets of interrelated KKT conditions

2.4 Mixed-Integer Linear Programming (MILP)

The MPEC model depicted in (11) – (20) is nonlinear on account of the slack complementarity constraints (15) – (20). These slack complementarity constraints are compactly written as $0 \le \mathrm{F}(\mathrm{x}) \perp \mathrm{x} \ge 0$, which is stated equivalently in vector form as:

$$\mathrm{F}(\mathrm{x}) \ge 0,\ \mathrm{x} \ge 0,\ \mathrm{F}(\mathrm{x})^{\mathrm{T}}\,\mathrm{x} = 0 \quad (21)$$

With the method in [10], however, this MPEC problem can be converted to a mixed-integer linear programming (MILP). The MILP model is presented as follows:

$$\text{Objective function: } (8) \quad (22)$$

s.t.

$$\text{Constraints in (12), (13) and (14)} \quad (23)$$

$$0 \le \mu_l^{s,\min} \le M_\mu^{\min} \nu_{\mu,l}^{s,\min} \quad (24)$$

$$\begin{aligned} 0 &\le Limit_l + \sum_{i=1}^{N} GSF_{l-i}\left(P_{Gi}^s + P_{Wi}^s - P_{Di}\right) \\ &\le M_\mu^{\min}\left(1 - \nu_{\mu,l}^{s,\min}\right) \end{aligned} \quad (25)$$

$$0 \le \mu_l^{s,\max} \le M_\mu^{\max} \nu_{\mu,l}^{s,\max} \quad (26)$$

$$\begin{aligned} 0 &\le Limit_l - \sum_{i=1}^{N} GSF_{l-i}\left(P_{Gi}^s + P_{Wi}^s - P_{Di}\right) \\ &\le M_\mu^{\max}\left(1 - \nu_{\mu,l}^{s,\max}\right) \end{aligned} \quad (27)$$

$$0 \le \omega_i^{s,\min} \le M_\omega^{\min} \nu_{\omega,\mathrm{i}}^{s,\min} \quad (28)$$

$$0 \le P_{Gi}^s - P_{Gi}^{\min} \le M_\omega^{\min}\left(1 - \nu_{\omega,i}^{s,\min}\right) \quad (29)$$

$$0 \le \omega_i^{s,\max} \le M_\omega^{\max} \nu_{\omega,\mathrm{i}}^{s,\max} \quad (30)$$

$$0 \le P_{Gi}^{\max} - P_{Gi}^{s} \le M_{\omega}^{\max}\left(1-\nu_{\omega,i}^{s\max}\right) \quad (31)$$

$$0 \le \varphi_{i}^{s,\min} \le M_{\varphi}^{\min}\nu_{\varphi,i}^{s,\min} \quad (32)$$

$$0 \le P_{Wi}^{s} \le M_{\varphi}^{\min}\left(1-\nu_{\varphi,i}^{s,\min}\right) \quad (33)$$

$$0 \le \varphi_{i}^{s,\max} \le M_{\varphi}^{\max}\nu_{\varphi,i}^{s,\max} \quad (34)$$

$$0 \le P_{Wi}^{s,\max} - P_{Wi}^{s} \le M_{\varphi}^{\max}\left(1-\nu_{\varphi,i}^{s\max}\right) \quad (35)$$

where $M_{\mu}^{\min}, M_{\mu}^{\max}, M_{\omega}^{\min}, M_{\omega}^{\max}, M_{\varphi}^{\min}, M_{\varphi}^{\max}$ are large enough constants and $\nu_{\mu,l}^{s,\min}, \nu_{\mu,l}^{s,\max}, \nu_{\omega,i}^{s,\min}, \nu_{\omega,i}^{s,\max}, \nu_{\varphi,i}^{s,\min}, \nu_{\varphi,i}^{s,\max}$ are the auxiliary binary variables [14].

3. Results and discussions

In this section, the bi-level optimization approach is performed on the modified PJM 5-bus system [13] and IEEE 24-bus system [16]. The MILP problem is solved by CPLEX 12.7 [17] under MATLAB environment.

The demand follows a normal distribution. The forecast mean value of demand is determined according to the data of test system and the standard deviation equals 10% from the mean. These test systems include two wind farms and the different scenarios for these wind power plants are given in Table 1.

Table 1. The uncertain scenarios for wind generation

Scenario	$P_{W1}^{s,\max}$ (MW	$P_{W2}^{s,\max}$ (MW)	Probability
1	200	200	0,04
2	200	360	0,16
3	360	200	0,16
4	360	360	0,64

When the future wind power production (no uncertainty) is perfectly known, it coincides with its expected value, given by 200. (0,04 + 0,16) + 360. (0,16 + 0,64) = 328 MW.

3.1 PJM 5-bus test system

The test system is modified from the PJM 5-bus system [13], as shown in Figure 3. Two wind plants (WF1 and WF2) are added into the system at buses A and C while one original generator is removed from bus A. The forecast mean load total is 1200 MW equally distributed among buses B, C and D.

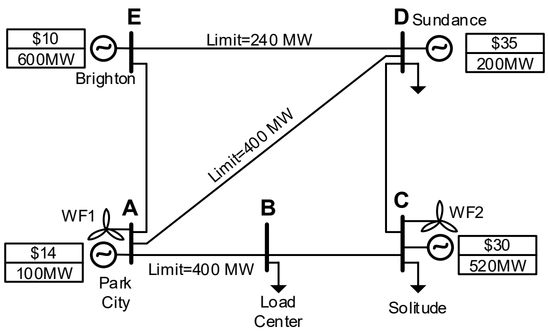

Figure 3. PJM 5-bus system with two wind farms

Table 2 shows LMP results achieved across all buses for two different cases: with uncertainty and without uncertainty. It should be emphasized that the findings calculated in this work are exactly the same in comparison with the MCS method (with 10000 samples), which is shown in Table 3. However, the simulation time (3.4 s) for bi-level optimization-based approach is dramatically lower than that of MCS (59,5 s).

Table 2. LMP results for PJM 5-bus system

Bus	Joint uncertainty of wind generation and demand	No uncertainty
A	[13.22, 15.83]	14.00
B	[14.00, 26.83]	19.39
C	[14.00, 29.01]	21.47
D	[14, 35]	27.17
E	[10, 14]	10.00

Table 3. LMP result intervals from MCS method and Bi-level optimization method

Bus	Bi-vel optimization method	MCS method
A	[13.22, 15.83]	[13.22, 15.83]
B	[14.00, 26.83]	[14.00, 26.83]
C	[14.00, 29.01]	[14.00, 29.01]
D	[14, 35]	[14, 35]
E	[10, 14]	[10, 14]

3.2 IEEE 24-bus test system

The test system is modified from the IEEE 24-bus system [16]. This system is used to further validate the effectiveness and robustness of the proposed approach. Two wind plants (WF1 and WF2) are added into the system at buses 7 and 8. The calculated results are illustrated as Figure 4. Moreover, MCS and bi-level optimization approach provides similar results.

4. Conclusions

This paper presents an approach to determine the intervals of locational marginal prices (LMPs) based on bi-level optimization model. Moreover, authors also present the conversion of this model to a mathematical program with equilibrium constraints (MPEC), then to a mixed-integer linear programming (MILP), which can be easily solved by available software tools. The results of this bi-level optimization problem reveal that the joint uncertainty of wind generation and the demand have a remarkable impact to LMP intervals. In the computational aspect, the bi-level optimization-based method is more efficient compared to Monte-Carlo simulations although the calculated results using both approaches are identical.

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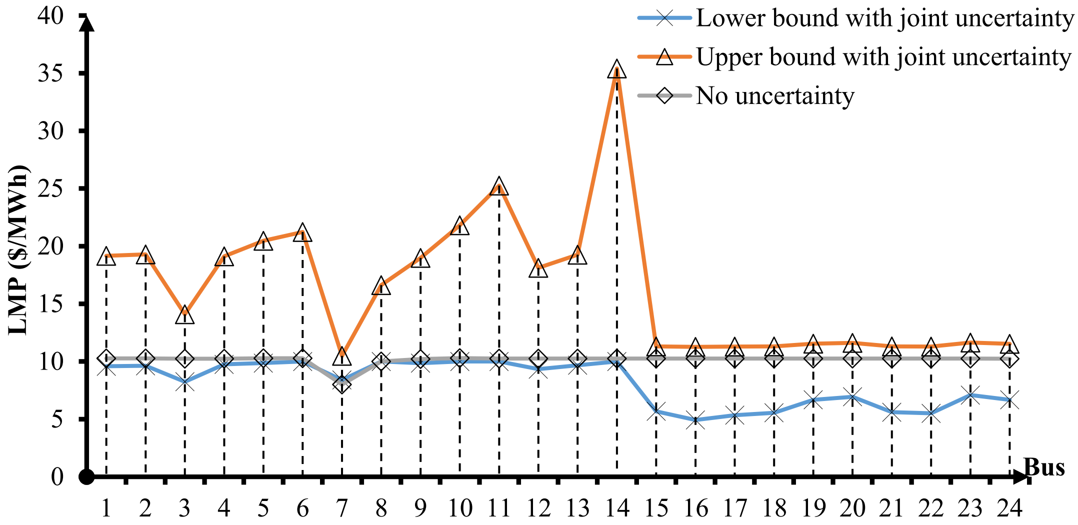

Fig. 4. LMP results for IEEE 24-bus system

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