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A Development toward Matching Pursuit Algorithm Aims To Reduce Calculation Mass in the Process of the Compessed Sampling and Errors in the Signal Recovery Process

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Abstract

Many works related to the signal restoration from discrete samples that are unsatisfactory requirements of Nyquist's criteria has been deployed and published recently. There were many algorithms to solve this issue, such as CoSaMP and the matching pursuit. This article presents a developed algorithm which is based on the matching pursuit algorithm to recover multi-dimensional architectures. This development is permissible to reduce the calculation mass in several cases along with a number of dedicated conditions when sampling signal compression. Simultaneously, the article will demonstrate this algorithm also makes reduce the occurred error at each step in the signal recovery process by using the mathematical method.

Keywords: Signal restoration, Matching pursuit, Compressed sampling

1. Introduction

Recently, related works to the improvement of the discrete signal recovery has been deployed in many published papers. One of most regarded important problems is signal restoration from discrete samples that are unsatisfactory requirements of Nyquist's criteria, also known as compressed sampling.

To recover the discrete signal, problems and related works have been given generally to become a loss function minimization problem under the conditions of discretized signal:

$$\min_x \ell(x) \text{ , with the condition of } \|x\|_{0,D} \le k \quad (1)$$

Here, $\|x\|_{0,D}$ is a standard matrix, this standard will be used to measure discretion of x relate to the set of D,$k$is a parameter to determine the discretion level of estimation $\ell(x)$ where it is consider as a targeting function and it is used to measure correlation between measurements and parameters need to estimate.

+ $\ell(x)$: is a smooth function, but needn't necessary to be a convex function. The estimation problem is the relative enough to cover related issues. For example, in the sensing data has a compression and discrete linear regression if we want to restore a discrete data $x^* \in \mathbb{R}^n$ from the discrete linear data $y \in \mathbb{R}^n : y = Ax^* + e$, with A is the measured matrix having the size of $m\times n$ and e is an interference vector.

The function $\ell(x)$ was established as a quadratic function: $\ell(x) = \|y - Ax\|_2^2$ following as the regulation of ℓ_0.

Similarly, in the recovery vector to restore a low level matrix X^*, the linear observation function $y = Ax^* + e$, loss function is a quadratic function $\|y - Ax\|_2^2$ and current limitations are the order of the matrix.

There are many algorithms related to the resolve of this issue, such as CoSaMP [2], [3] and the matching pursuit [4]. In which, the matching pursuit algorithm has some advantages in comparison with CoSaMP algorithm.

2. Matching pursuit algorithm

Input: k and stop condition.

Initialize: $\Lambda = \varnothing$, x^0, và $t = 0$.

Loop:

Assign: $u = \nabla\ \ell(x^t)$

Identify: $\Gamma = \operatorname{argmax}_{\Omega}\{\ \|P_{D_\Omega} u\|_2 : |\Omega| \le 2k\}$.

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Combine: $\hat{\Gamma} = \Gamma \cup \Lambda$

Estimate: $b = \text{argmin}_x \; \ell(x) \quad \text{s.t.} \quad x \in C_{D_{\hat{\Gamma}}}$

Remove: $\Lambda = \text{argmax}_{\Omega}\{ \; \|P_{D_\Omega} b\|_2 : |\Omega| \leq k\}$

Update: $x^{t+1} = P_{D_\Lambda} b$

t = t + 1

Until: Right halt criterion

Output: x^t

However, in the matching pursuit algorithm according to the gradient at identified and removed steps, this algorithm needs to find a subset *Γ* of the columns in the set *D* satisfying the orthogonal isometry conditions of one vector onto the extension set (*DΓ*) that achieves the largest energy. This subset is usually difficult to compute, especially when D is a fully completed set with an ultra-large number of elements. The reason is the need to seek out in all possible combinations of subsets of *D* to find out the best option [5].

In fact, to overcome the limitations above needing to pay attention to special properties of the set *D*. For example, when *D* is orthogonal, *Γ* can be selected simply by isometry*u* onto space expanded by all the columns of the *D* and select the largest items of *2k* from this isometry. In addition, when doing the restore low-level matrix, *U* is a matrix and *Γ* can be calculated by taking the singularity value degree (SVD) of *U* and selecting the left and right singularity vectors to the best *2k* related to the peculiar value of the largest *2k* [4]. In the next section, the article proposes an improved matching pursuit algorithm in order to reduce the calculation mass in the compressed sampling process and reduce errors that occur at each step in the process of signal recovery on the basis of considering some dedicated conditions for the set *D*.

3. Development algorithm of matching pursuit

The presented algorithm will be simpler than algorithm in the section 1. However, compared to the original algorithm, improved matching pursuit algorithm requires several certain conditions of the set *D* so that the algorithm may converge, so called DRMP algorithm (D-RIP Matching Pursuit). Suppose that *D* is finite, and *D* complies with rules of RIP (Restricted isometry property) [7]. More specifically, *D* satisfies RIP if in there exists a constant $\delta_k \in [0,1)$ for satisfying:

$$(1-\delta_k)\|\alpha\|_2^2 \leq \|D\alpha\|_2^2 \leq (1+\delta_k)\|\alpha\|_2^2 \qquad (1)$$

for each mainly vector α at k. When *D* is the orthogonal matrix with the size as , , and the alternative algorithm would be equivalent to the original algorithm.

Algorithm DRMP:

Input: *k* and halt condition

Initialize: $\Lambda = \emptyset$ **,** x^0**, và** $t = 0$**.**

Loop:

Assign: $u = \nabla \; \ell(x^t)$

Identify: $\Gamma = \text{argmax}_{\Omega}\{ \; \|P_{D_\Omega} u\|_2 : |\Omega| \leq 2k\}$

Combine: $\hat{\Gamma} = \Gamma \cup \Lambda$

Estimate: $b = \text{argmin}_x \; \ell(x) \quad \text{s.t.} \quad x \in C_{D_{\hat{\Gamma}}}$

Remove: $\Lambda = \text{argmax}_{\Omega}\{ \; \|P_{D_\Omega} b\|_2 : |\Omega| \leq k\}$

Update: $x^{t+1} = P_{D_\Lambda} b$

t = t + 1

Until: Halt right criterion

Output: x^t

DRMP algorithm is almost equivalent to the initial matching pursuit algorithm. The difference between two algorithms is at the identified step and the removed step. In which, we replaced the isometry by its inner multification. The following theorem indicates that, for this improved algorithm, restoration errors of the signal will decrease gradually after each of iteration and this calculation will be simplified the calculation for the orthogonal isometry. However, this can be only applied when the matrix D satisfies the conditions of RIP. When D is the orthogonal matrix, two algorithms are equivalent.

Theorem:

Assume that $\hat{x}$ as a result of the expression (1) and $\epsilon(\hat{x}) \triangleq \max_i | \langle \nabla\ell(\hat{x}), d_i \rangle |$, estimated errors at the iteration with the order (t+1) be bound by:

$$\|x^{t+1} - \hat{x}\|_2 \leq \varphi \; \|x^t - \hat{x}\|_2 + \epsilon(\hat{x})\sqrt{k}\left(\frac{\sqrt{2}+1}{\tau^-_{4k}} + \frac{3}{2\sqrt{\tau^-_{4k}\tau^+_{4k}}}\right) \qquad (2)$$

Here, φ is a microdegration rate and $\varphi \triangleq \sqrt{2\frac{1+\delta}{(1-\delta)^2}\frac{\tau^+_k((1+\delta)\tau^+_{2k} - (1-\delta)\tau^-_{2k})}{(\tau^-_{4k})^2}}$

4. Theorem provement

We denoted x to be x_R if x can be presented by the formula $x=\sum_{i\in R}\alpha_i d_i$. α_i is the factor of x in the set D Assume that a possible root $\hat{x}$ of (1) is T.

Lemma 1

Assume that b is a vector obtaining from estimated step in the replacement algorithm at the iteration order t-th, and denote $\lambda \triangleq \max_i \left|\langle \nabla\ \ell(\hat{x}), d_i\rangle\right|$. Denote $\hat{\Gamma}$ is the achieved set at the combined step order t-th. Analyze $\hat{x}=\hat{x}_{T\setminus\hat{\Gamma}}+\hat{x}_{T\cap\hat{\Gamma}}$ then we have:

$$\|b-\hat{x}\|_2 \le \sqrt{\frac{\rho_k^+}{\rho_{4k}^-}}\ \|\hat{x}_{T\setminus\hat{\Gamma}}\|_2+\frac{5\lambda\sqrt{k}}{2\rho_{4k}^-\sqrt{1-\delta_{4k}}} \tag{3}$$

Lemma 2

Denote R to be the support of the vector $\Delta \triangleq x^t-\hat{x}$ to the set D and $\lambda \triangleq \max_i \left|\langle \nabla\ \ell(\hat{x}), d_i\rangle\right|$, we have:

$$\sqrt{\frac{\rho_{2k}^+-\rho_{2k}^-}{2\rho_{4k}^-}}\|x^{t-1}-\hat{x}\|_2+\frac{\lambda\sqrt{2k}}{\rho_{4k}^-}\|\Delta_{R\setminus\Gamma}\|_2 \le$$

$$\sqrt{\frac{1+\delta}{(1-\delta)^2}\frac{(1+\delta)\rho_{2k}^+-(1-\delta)\rho_{2k}^-}{2\rho_{4k}^-}}\|\Delta\|_2 \tag{4}$$

$$+\lambda\sqrt{k}\frac{1+\delta}{1-\delta}\left(\frac{1}{\sqrt{2\rho_{4k}^-}}+\frac{1}{\sqrt{2\rho_{4k}^-}}\right)$$

With two lemmas above, the theorem can be proved as follow.

We have:

$$\|x^{t+1}-\hat{x}\|_2 \le \|b-\hat{x}\|_2+\|b-x^{t+1}\|_2 \le 2\|b-\hat{x}\|_2 \tag{5}$$

Use the lemma 1, from the algorithm procedure, the support $\lambda \triangleq \max_i \left|\langle \nabla\ \ell(\hat{x}), d_i\rangle\right| rt)$ of x^t belongs to the set $\hat{\Gamma}$. Thus, $x^t_{T\setminus\hat{\Gamma}}=0$ and we also get:

$$\|\hat{x}_{T\setminus\hat{\Gamma}}\|_2=\|(\hat{x}-x^t)_{T\setminus\hat{\Gamma}}\|_2 \le \|(\hat{x}-x^t)_{T\setminus\Gamma}\|_2$$

$$= \|(\hat{x}-x^t)_{R\setminus\Gamma}\|_2 \tag{6}$$

Here, R is a symbol of the support $x^t-\hat{x}$

The first inequality obtains from $\Gamma\subset\hat{\Gamma}$, thus $T\setminus\hat{\Gamma}\subset T\setminus\Gamma$. The second inequality obtains from $T\subset R$. Apply the lemma 2, we will be obtained the thing that must be proved.

Clause:

Suppose that x is a vector k-sparse in the set D with the value T, we denote as $x=D_T\alpha_T$. For any vector y has a size of n, we have:

$$|\langle x,y\rangle| \le \max_{i\in T}|\langle d_i,y\rangle|\frac{\sqrt{k}}{\sqrt{(1-\delta_k)}}\ \|x\|_2 \tag{7}$$

The clause will be proved as follow.

We have:

$$|\langle x,y\rangle| = \left|\left\langle\sum_{i\in T}\alpha_i d_i, y\right\rangle\right| = \left|\sum_{i\in T}\langle\alpha_i d_i, y\rangle\right| \le \sum_{i\in T}|\alpha_i|\,|\langle d_i,y\rangle|$$

$$=\max_{i\in T}|\langle d_i,y\rangle|\ \|\alpha_T\|_1 \le \max_{i\in T}|\langle d_i,y\rangle|\sqrt{k}\ \langle\alpha_T\rangle_2$$

$$\le \max_{i\in T}|\langle d_i,y\rangle|\sqrt{k}\frac{1}{\sqrt{(1-\delta_k)}}\ \|D_T\alpha_T\|_2 \tag{8}$$

Where, the last inequality is following the condition RIP of the set D.

Prove the lemma 1.

For desmonstrating the lemma 1, apply condition D- RSC (D- restricted strong convexity) and D- RSS (D- restricted strong smoothness) [6], [8] and the clause above, we get:

$$\rho_{4k}^-\ \|b-\hat{x}\|_2^2\ +\ \langle\nabla\ \ell(\hat{x}), b-\hat{x}\rangle \le^{(a)}\ \ell(b)-\ \ell(\hat{x})$$

$$\le^{(b)}\ \ell(\hat{x}_{T\cap\hat{\Gamma}})-\ \ell(\hat{x})$$

$$=\ell(\hat{x}_{T\cap\hat{\Gamma}})-\ \ell(\hat{x})-\langle\nabla\ell(\hat{x}),\hat{x}_{T\cap\hat{\Gamma}}-\hat{x}\rangle+\langle\nabla\ \ell(\hat{x}),\hat{x}_{T\cap\hat{\Gamma}}-\hat{x}\rangle$$

$$\le^{(c)}\ \rho_k^+\ \|\hat{x}_{T\cap\hat{\Gamma}}-\hat{x}\|_2^2+\langle\nabla\ \ell(\hat{x}),\hat{x}_{T\setminus\hat{\Gamma}}\rangle$$

$$\le^{(d)}\ \rho_k^+\ \|\hat{x}_{T\setminus\hat{\Gamma}}\|^2+\max_{i\in T\setminus\hat{\Gamma}}\left|\langle\nabla\ \ell(\hat{x}),d_i\rangle\right|\frac{\sqrt{k}}{\sqrt{1-\delta_k}}\ \|\hat{x}_{T\setminus\hat{\Gamma}}\|_2$$

$$=\rho_k^+\ \|\hat{x}_{T\setminus\hat{\Gamma}}\|_2^2+\frac{\lambda\sqrt{k}}{\sqrt{1-\delta_k}}\ \|\hat{x}_{T\setminus\hat{\Gamma}}\|_2$$

$$\le\rho_k^+\left(\ \|\hat{x}_{T\setminus\hat{\Gamma}}\|_2+\frac{\lambda\sqrt{k}}{2\rho_k^+\sqrt{1-\delta_k}}\right)^2 \tag{9}$$

The inequality (a) achieves from supposed condition D- RSC; the inequality (b) achieves following the algorithm, $\ell(b)$ is a minimization point $\ell(x)$ for $x\in\operatorname{span}(D_{\hat{\Gamma}})$; the inequality (c) achieves from assumed conditions of D- RSS; finally, the inequality (d) is from the referred clause above. Simultaneously, we have:

$$|\operatorname{supp}_D(b-\hat{x})| \le |\hat{\Gamma}\cup T| \le 4k$$

the left item of the inequality can be limited by applying the clause above:

$$\rho^{-}_{4k}\left\|b-\hat{x}\right\|_2^2+\left\langle \nabla\ \ell(\hat{x}), b-\hat{x}\right\rangle$$
$$\geq \rho^{-}_{4k}\ \left\|b-\hat{x}\right\|_2^2-\ell\ \frac{\sqrt{4k}}{\sqrt{1-\delta_{4k}}}\ \left\|b-\hat{x}\right\|_2$$
$$\geq \rho^{-}_{4k}\left(\ \left\|b-\hat{x}\right\|_2-\frac{\ell\sqrt{4k}}{2\rho^{-}_{4k}\sqrt{1-\delta_{4k}}}\right)^2-\frac{\ell^{\,2}k}{\rho^{-}_{4k}(1-\delta_{4k})}\quad (10)$$

Combine inequalities above, we obtain:

$$\left(\left\|b-\hat{x}\right\|_2-\frac{\ell\sqrt{k}}{\rho^{-}_{4k}\sqrt{1-\delta_{4k}}}\right)^2\leq$$
$$\frac{\rho^{+}_{k}}{\rho^{-}_{4k}}\left(\left\|\hat{x}_{T\backslash\hat{\Gamma}}\right\|_2+\frac{\ell\sqrt{k}}{2\rho^{+}_{k}\sqrt{1-\delta_{k}}}\right)^2+\frac{\ell^{\,2}k}{(\rho^{-}_{4k})^2(1-\delta_{4k})}\quad (11)$$

Deduce:

$$\left\|b-\hat{x}\right\|_2-\frac{\ell\sqrt{k}}{\rho^{-}_{4k}\sqrt{1-\delta_{4k}}}\leq$$
$$\sqrt{\frac{\rho^{+}_{k}}{\rho^{-}_{4k}}}\left(\left\|\hat{x}_{T\backslash\hat{\Gamma}}\right\|_2+\frac{\ell\sqrt{k}}{2\rho^{+}_{k}\sqrt{1-\delta_{k}}}\right)+\frac{\ell\sqrt{k}}{\rho^{-}_{4k}\sqrt{(1-\delta_{4k})}}.$$
$$\left\|b-\hat{x}\right\|_2\leq\sqrt{\frac{\rho^{+}_{k}}{\rho^{-}_{4k}}}\ \left\|\hat{x}_{T\backslash\hat{\Gamma}}\right\|_2+\frac{5\ell\sqrt{k}}{2\rho^{-}_{4k}\sqrt{1-\delta_{4k}}}\quad (12)$$

Prove the lemma 2

As denoted $\Delta=\hat{x}-x^t$ and $\mathrm{supp}_D(\Delta)=R$ we have $\Delta=\sum_{i\in R}\alpha_i d_i=\Delta_{R\cap\Gamma}+\sum_{i\in R\backslash\Gamma}\alpha_i d_i$. By applying the condition D-RSC for x^t, we have:

$$\ell(x^t+\Delta)-\ \ell(x^t)-\rho^{-}_{2k}\ \left\|\Delta\right\|_2^2\ \geq\ \left\langle\nabla\ \ell(x^t),\Delta\right\rangle$$
$$=\ \left\langle\nabla\ \ell(x^t),\sum_{i\in R}\alpha_i d_i\right\rangle$$
$$=\ \left\langle\nabla\ \ell(x^t),\Delta_{R\cap\Gamma}\right\rangle+\sum\nolimits_{i\in R\backslash\Gamma}\left\langle\nabla\ \ell(x^t),\alpha_i d_i\right\rangle$$
$$\geq\ \left\langle\nabla\ell(x^t),\Delta_{R\cap\Gamma}\right\rangle-\sum\nolimits_{i\in R\backslash\Gamma}|\alpha_i|\left|\ \left\langle\nabla\ell(x^t),d_i\right\rangle\right|\quad (13)$$

It is clear that features of the subset R are smaller than $2k$, while $|\Gamma|=2k$. This is deduce to $|R\backslash\Gamma|\leq|\Gamma\backslash R|$. Combined with the construction of the subset Γ, we have:

$$\left|\left\langle\nabla\ell(x^t),d_i\right\rangle\right|\leq\left|\left\langle\nabla\ell(x^t),d_j\right\rangle\right|,$$

for $i\in R\backslash\Gamma$ and $j\in\Gamma\backslash R$. Let's put the vector $g\in\mathbb{R}^p$ including the factors to satisfy:

- Factors is out of the set $\Gamma\backslash R$ then is set equally 0.
- For each item $i\in R\backslash\Gamma$, we select a item $j\in\Gamma\backslash R$ and put:

$$g_j=-|\alpha_i|\frac{\left\langle\nabla\ell(x^t),d_i\right\rangle}{\left|\ \left\langle\nabla\ell(x^t),d\right\rangle_i\right|},$$

- Factors in the set $\Gamma\backslash R$ without being chosen by steps above then are set equally zero. Có tối đa $|\Gamma\backslash R|-|R\backslash\Gamma|$ hệ số tại bước này.

So we obtain:

$$\sum\nolimits_{i\in R\backslash\Gamma}\alpha_i^2=\sum\nolimits_{j\in\Gamma\backslash R}g_j^2.\quad (14)$$

For each of a numerical couple $i\in R\backslash\Gamma$ and $j\in\Gamma\backslash R$ that chose at the step 2, we have: $|g_j|=|\alpha_i|$. Deduce:

$$-|\alpha_i|\left|\ \left\langle\nabla\ell(x^t),d_i\right\rangle\right|\geq-|\alpha_i|\left|\left\langle\nabla\ell(x^t),d_j\right\rangle\right|$$
$$=\left\langle\nabla\ \ell(x^t),g_j d_j\right\rangle\quad (15)$$

Here, the second uniformity expression complies the construction from $g_{\Gamma\backslash R}$. Thus we have:

$$-\sum\nolimits_{j\in R\backslash\Gamma}\ \left\langle\nabla\ \ell(x^t),g_j d_j\right\rangle\geq\sum\nolimits_{j\in\Gamma\backslash R}\ \left\langle\nabla\ \ell(x^t),g_j d_j\right\rangle\quad (16)$$

Combine with (13) we have:

$$\ell(x^t+\Delta)-\ell(x^t)-\rho^{-}_{2k}\left\|\Delta\right\|_2^2$$
$$\geq\ \left\langle\nabla\ell(x^t),\Delta_{R\cap\Gamma}\right\rangle+\sum\nolimits_{j\in\Gamma\backslash R}\ \left\langle\nabla\ \ell(x^t),g_j d_j\right\rangle$$
$$=\ \left\langle\nabla\ell(x^t),\Delta_{R\cap\Gamma}\right\rangle+\left\langle\nabla\ \ell(x^t),\sum\nolimits_{j\in\Gamma\backslash R}g_j d_j\right\rangle$$
$$=\ \left\langle\nabla\ \ell(x^t),z\right\rangle\quad (17)$$

Here, we set $z\triangleq\Delta_{R\cap\Gamma}+\sum_{j\in\Gamma\backslash R}g_j d_j$. In which: z is a discrete vector of the set D and its value is Γ. The right item of the inequality can be upper limited D-RSC.

$$\left\langle\nabla\ \ell(x^t),z\right\rangle\ \geq\ \ell(x^{t-1}+z)-\ \ell(x^t)-\rho^{+}_{2k}\left\|z\right\|_2^2\quad (18)$$

Thus we have:

$$\rho^{+}_{2k}\ \left\|z\right\|_2^2-\rho^{-}_{2k}\ \left\|\Delta\right\|_2^2\ \geq\ \ell(x^t+z)-\ \ell(x^{t-1}+\Delta)$$
$$=\ \ell(x^t+z)-\ell(x^*)\quad (19)$$

From the definition of z and the RIP condition of the set D:

$$\|z\|_2^2 = \left\|\sum_{R\cap\Gamma}\beta_i d_i + \sum_{j\in\Gamma\backslash R} g_j d_j\right\|_2^2$$

$$\le (1+\delta_{2k})\left(\sum_{R\cap\Gamma}\beta_i^2 + \sum_{j\in\Gamma\backslash R} g_j^2\right)$$

$$= (1+\delta_{2k})\left(\sum_{R\cap\Gamma}\beta_i^2 + \sum_{i\in R\backslash\Gamma}\beta_i^2\right) = (1+\delta_{2k})\left(\sum_{i\in R}\beta_i^2\right)$$

$$\le \frac{1+\delta_{2k}}{1-\delta_{2k}}\left\|\sum_{i\in R}\beta_i d_i\right\|_2^2 = \frac{1+\delta_{2k}}{1-\delta_{2k}}\|\Delta\|_2^2 \qquad (20)$$

Therefore, the left item of the expression (19) has been limited by:

$$\rho_{2k}^{+}\|z\|_2^2 - \rho_{2k}^{-}\|\Delta\|_2^2 \le \left(\frac{1+\delta_{2k}}{1-\delta_{2k}}\rho_{2k}^{+} - \rho_{2k}^{-}\right)\|\Delta\|_2^2 \qquad (21)$$

The right item can be limited lower by D - RSC.

$$\ell(x^t+z) - \ell(x^\star) \ge \langle\nabla\ell(\hat{x}), z-\Delta\rangle + \rho_{4k}^{-}\|z-\Delta\|_2^2 \qquad (22)$$

Note that: $\mathrm{supp}_D(z)=\Gamma$ and $\mathrm{supp}_D(\Delta)=R$,

we obtain:

$\mathrm{supp}_D(z-\Delta)\in R\cap\Gamma$ and $|\mathrm{supp}_D(z-\Delta)|\le 4k$.

Denote that $z-\Delta = D_{R\cap\Gamma}\alpha_{R\cap\Gamma} = \sum_{i\in R\cap\Gamma} d_i\alpha_i$, we have:

$$\ell(x^t+z) - \ell(x^\star) \ge -\sum_{i\in R\cap\Gamma}\alpha_i\langle\nabla L(x^\star), d_i\rangle + \rho_{4k}^{-}\|z-\Delta\|_2^2$$

$$\ge -\sum_{i\in R\cap\Gamma}|\alpha_i||\langle\nabla L(x^\star), d_i\rangle| + \rho_{4k}^{-}\|z-\Delta\|_2^2$$

$$\ge -\ell\sum_{i\in R\cap\Gamma}|\alpha_i| + \rho_{4k}^{-}\|z-\Delta\|_2^2$$

$$\ge -\ell\sqrt{4k}\|\alpha_{R\cap\Gamma}\|_2 + \rho_{4k}^{-}\|z-\Delta\|_2^2$$

$$\ge -\ell\sqrt{4k}\frac{1}{\sqrt{1-\delta_{4k}}}\|D_{R\cap\Gamma}\alpha_{R\cap\Gamma}\|_2 + \rho_{4k}^{-}\|z-\Delta\|_2^2$$

$$\ge -\frac{\ell\sqrt{4k}}{\sqrt{1-\delta_{4k}}}\|z-\Delta\|_2 + \rho_{4k}^{-}\|z-\Delta\|_2^2$$

$$= \rho_{4k}^{-}\left(\|z-\Delta\|_2 - \frac{\ell\sqrt{k}}{\rho_{4k}^{-}\sqrt{1-\delta_{4k}}}\right)^2 - \frac{\ell^2 k}{\rho_{4k}^{-}} \qquad (23)$$

From constructing vectors z and g , and RIP conditions of D , we have:

$$\ge (1-\delta_{4k})\left(\sum_{j\in\Gamma\backslash R} g_j^2 + \sum_{i\in R\backslash\Gamma}\beta_i^2\right)\|z-\Delta\|_2^2$$

$$= \left\|\sum_{j\in\Gamma\backslash R} g_j d_j - \sum_{i\in R\backslash\Gamma}\beta_i d_i\right\|_2^2$$

$$= 2(1-\delta_{4k})\sum_{i\in R\backslash\Gamma}\beta_i^2 \ge 2\frac{1-\delta_{4k}}{1+\delta_{4k}}\left\|\sum_{i\in R\backslash\Gamma}\beta_i d_i\right\|_2^2$$

$$= 2\frac{1-\delta_{4k}}{1+\delta_{4k}}\|\Delta_{R\backslash\Gamma}\|_2^2 \qquad (24)$$

Here, the second equality complies with (14). Combine with above inequality, we have:

$$\ell(x^t+z) - \ell(x^\star) \ge \rho_{4k}^{-}\left(\sqrt{2\frac{1-\delta_{4k}}{1+\delta_{4k}}}\|\Delta_{R\backslash\Gamma}\|_2 - \frac{\ell\sqrt{k}}{\rho_{4k}^{-}\sqrt{1-\delta_{4k}}}\right)^2 - \frac{\ell^2 k}{\rho_{4k}^{-}} \qquad (25)$$

Combine result above with expressions (19) and (20), we have:

$$\left(\frac{1+\delta_{4k}}{1-\delta_{4k}}\rho_{2k}^{+} - \rho_{2k}^{-}\right)\|\Delta\|_2^2 \ge \rho_{4k}^{-}\left(\sqrt{2\frac{1-\delta_{4k}}{1+\delta_{4k}}}\|\Delta_{R\backslash\Gamma}\|_2 - \frac{\ell\sqrt{k}}{\rho_{4k}^{-}\sqrt{1-\delta_{4k}}}\right)^2 - \frac{\ell^2 k}{\rho_{4k}^{-}} \qquad (26)$$

Deduce:

$$\sqrt{2\frac{1-\delta_{4k}}{1+\delta_{4k}}}\|\Delta_{R\backslash\Gamma}\|_2 \le \frac{1}{\sqrt{\rho_{4k}^{-}}}\sqrt{\left(\frac{1+\delta_{4k}}{1-\delta_{4k}}\rho_{2k}^{+} - \rho_{2k}^{-}\right)\|\Delta\|_2^2 + \frac{\ell^2 k}{\rho_{4k}^{-}}} + \frac{\ell\sqrt{k}}{\rho_{4k}^{-}\sqrt{1-\delta_{4k}}}$$

$$\le \sqrt{\frac{1}{\rho_{4k}^{-}}\left(\frac{1+\delta_{4k}}{1-\delta_{4k}}\rho_{2k}^{+} - \rho_{2k}^{-}\right)}\|\Delta\|_2 + \frac{2\ell\sqrt{k}}{\rho_{4k}^{-}\sqrt{1-\delta_{4k}}} \qquad (27)$$

Deduce:

$$\|\Delta_{R\backslash\Gamma}\|_2 \le \sqrt{\frac{1+\delta}{(1-\delta)^2}\frac{(1+\delta)\rho_{2k}^{+} - (1-\delta)\rho_{2k}^{-}}{2\rho_{4k}^{-}}}\|\Delta\|_2 + \ell\sqrt{k}\frac{1+\delta}{1-\delta}\left(\frac{1}{\sqrt{2\rho_{4k}^{-}}} + \frac{1}{\sqrt{2\rho_{4k}^{-}}}\right) \qquad (28)$$

The last, the theorem has been demonstrated with the finite condition of D which complies with RIP property. For the new proposed DRMP algorithm, the theorem shows that signal recovery errors will be reduced gradually after each of iteration and simply for the calculation.

5. Conclusion

The article has presented a development of matching pursuit algorithm which is based on an original matching pursuit algorithm to restore multidimensional structures. The difference between the two algorithms is the identified step and the removed step. Compared with conventional matching pursuit algorithm, this algorithm is simpler than in a number of cases, especially in cases of low rank matrix restoration. Meanwhile, this algorithm can reduce the number of calculations with some certain conditions of the signal. Simultaneously, the new proposed algorithm would reduction at each error in the signal recovery process.

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