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Centralized Optimal Control with a Shared Signal for Identical Subsystems: Application to Multiple Four-Tank Systems

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Abstract

This study proposes a centralized control solution aimed at reducing implementation costs for industrial systems comprising multiple similar subsystems. Instead of equipping each actuator to execute separate control signals, the proposed method utilizes a shared control signal to simultaneously operate multiple independent systems that share identical setpoints and technical specifications. Analytical results demonstrate that this control architecture simplifies the hardware structure and reduces the required control resources, thereby lowering investment and maintenance costs. However, the use of a common control signal introduces certain trade-offs in control performance, such as increased delay and greater liquid level oscillation compared to independently controlled systems. To validate the approach, we conducted simulations on multiple clusters of four-tank experimental models under various initial conditions. The simulation results confirm that the proposed method ensures system stability and effective setpoint tracking. These findings suggest a promising direction for the development of centralized control architectures in large-scale or multi-agent systems.

Keywords: Centralized optimal control, four-tank benchmark, large-scale systems, shared control input.

1. Introduction

In the context of industrial modernization and the growing demand for automation, large-scale control systems are increasingly facing major challenges related to performance optimization, scalability, and cost management. Such systems, often referred to as large-scale systems [1], are composed of multiple subsystems or units operating in parallel (possibly interacting or decoupled) and are typically distributed across space. Controlling these systems requires not only effective coordination among the subsystems but also careful consideration of constraints on computational resources, monitoring capability, and fault tolerance in the presence of local failures.

Traditional control architectures are typically categorized into two main types: centralized control [2] and decentralized/distributed control [3]. While decentralized architectures are popular in large systems due to their modularity, robustness to local faults, and ease of integration, they often incur high hardware costs, synchronization complexity, and limitations in achieving global optimality. In contrast, the Centralized Control System (CCS) benefits from full access to state information across all control loops, enabling the design of globally optimized control strategies at a single point, and significantly simplifying supervision, tuning, and maintenance. In distributed and multi-agent frameworks, synchronization and coordination among agents are key aspects that determine overall system performance [4, 5]. These studies on cooperative control and fixed-time coordination further highlight the importance of maintaining consistent dynamic behavior across interacting subsystems, which closely relates to the motivation of the present work.

Notably, recent studies have explored trade-offs between control performance and implementation cost [6], particularly in practical industrial applications where computational power, communication infrastructure, and actuator availability are often limited. However, control frameworks that aim to reduce the number of actuators across structurally identical subsystems have not yet been fully explored, especially in large-scale systems with repetitive or homogeneous structures.

Recent studies have continued to emphasize the trade-offs between centralized and distributed control architectures in large-scale systems. It has been pointed out that while global optimality and simplicity are offered by centralized MPC, issues of scalability and reliability are encountered when such schemes are implemented in large-scale applications such as wind power networks [7]. Centralized control has been described as “simple, convenient, and easy to realize” yet its use has been constrained by high computational requirements and susceptibility to single-point failures [7]. It has also been highlighted that fully centralized control becomes impractical once the system scale exceeds a certain threshold, primarily due to communication and memory constraints [8]. To mitigate these challenges, hybrid control strategies

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(particularly those designed to reduce the number of control inputs) have been proposed as a practical balance between coordination and feasibility [8]. These observations have further reinforced the motivation for the development of a shared-signal centralized control framework, intended to integrate the global optimality of centralized control with the scalability and resource efficiency of distributed approaches.

Motivated by this context, this paper proposes a novel centralized control architecture, in which a single controller generates a shared control signal to simultaneously drive multiple subsystems with identical dynamics and structure. This approach leverages the homogeneity of the subsystems to reuse the control signal, thereby significantly reducing hardware costs, simplifying actuator design, and facilitating centralized tuning and upgrades. Although the subsystems operate independently in a physical sense, sharing a single control signal from a centralized controller can still ensure accurate reference tracking, system-wide stability, and coordinated dynamic behavior. This approach is particularly suitable for large-scale systems with repetitive structures and high degrees of symmetry, where cost-efficiency in deployment is a critical requirement.

To verify the feasibility and effectiveness of the proposed method, the four-tank system (a well-known benchmark in liquid level control) is employed as the experimental platform. This system features nonlinearities, multivariable interactions, and the ability to switch between minimum and non-minimum phase configurations, making it well-suited to emulate realistic industrial conditions. In this study, a linearized version of the four-tank model [9] is used and replicated into multiple identical subsystems, with the centralized controller designed to simultaneously control all units through a single shared control signal. The simulation results not only validate the correctness of the proposed architecture but also highlight its potential as a cost-effective control solution for homogeneous large-scale and multi-agent systems, while laying the foundation for future research in cooperative and resource-efficient distributed control.

The main contributions of this paper can be summarized as follows:

(i) A novel centralized control architecture is proposed, in which a single control signal is shared across multiple structurally identical subsystems, along with an extended mathematical model to support this architecture.

(ii) A theoretical result is developed and proved to guarantee the optimality, stability, and performance of the centralized controller for the entire interconnected system.

(iii) The effectiveness of the proposed method is evaluated through simulations on multiple four-tank system instances under various operating conditions.

The remainder of this paper is organized as follows. Section 2 presents the problem formulation and a standard centralized controller design. Section 3 introduces the proposed shared-signal centralized control approach, including a formal theorem and its detailed proof. In Section 4, the simulation results are presented, and the performance of the proposed controller is compared with traditional centralized strategies. Finally, Section 5 concludes the paper and outlines future research directions.

2. Centralized Controller Design

Consider a system consisting of a linear subsystems (satisfying controllability and observability properties) that do not interact with each other, in the following form:

$$\begin{cases} x_i(k+1) = A_i x_i(k) + B_i u_i(k) \\ y_i(k) = C_i x_i(k) \end{cases} \tag{1}$$

where $i = 1,2,\ldots,a$. $x_i(k) \in \mathbb{R}^n$: state vector of the i-th subsystem, $u_i(k) \in \mathbb{R}^m$: control input vector of the i-th subsystem, $y_i(k) \in \mathbb{R}^p$: output vector of the i-th subsystem, $A_i \in \mathbb{R}^{n\times n}$: state matrix, $B_i \in \mathbb{R}^{n\times m}$: input matrix, $C_i \in \mathbb{R}^{p\times n}$: output matrix.

It can be seen that the centralized controller design for (1) can be approached in two ways. The first is to individually solve a Riccati equations using methods such as those in [10] for each subsystem i, and then compute the controllers for each subsystem accordingly. The second is to combine all the state, input, and output matrices along with their corresponding variables into a single large-scale system, in which case the controller design requires solving a single large Riccati equation, similar to the approaches in [11]. Based on system (1), we formulate an LQI problem similar to [12] to track the reference signal $r_i(t)$ for each subsystem i.

Based on the formulas presented in [12], the tracking error is defined as:

$$e_i(k) = r_i(k) - y_i(k) = r_i(k) - C_i x_i(k) \tag{2}$$

From this, the accumulated tracking error over time is computed as:

$$z_i(k+1) = z_i(k) + e_i(k) \tag{3}$$

where, $z_i(k)$ represents the accumulated tracking error, i.e., the integral of the instantaneous error $e_i(k)$. This term introduces an integral state to eliminate steady-state tracking error in the control law.

By augmenting the state variables into $\tilde{x}_i = \begin{bmatrix} x_i^T(k) & z_i^T(k) \end{bmatrix}^T$, we obtain the following extended system:

$$\tilde{x}_i(k+1) = \tilde{A}_i \tilde{x}_i(k) + \tilde{B}_i u_i(k) + \tilde{E} r_i(k) \tag{4}$$

where

$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ -C_i & I_p \end{bmatrix} \in \mathbb{R}^{(n+p)\times(n+p)},\ \tilde{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix} \in \mathbb{R}^{(n+p)\times m},$$

$$\tilde{E} = \begin{bmatrix} 0 \\ I_p \end{bmatrix} \in \mathbb{R}^{(n+p)\times p}$$

This formulation corresponds to a Linear Quadratic Integral (LQI) control scheme, which augments the state vector with the integral of the tracking error. The LQI structure ensures zero steady-state error for constant reference inputs.

We consider a cost function to compute the optimal controller for system (4), similar to [13], where $Q_i \in \mathbb{R}^{n\times n}$ is the state weighting matrix and $R_i \in \mathbb{R}^{m\times m}$ is the control weighting matrix. By extending the weighting matrix to $\tilde{Q}_i = \operatorname{diag}(Q_i, O)$, where $O \in \mathbb{R}^{p\times p}$ denotes a zero matrix, to match the augmented system (4), the cost function becomes:

$$J = \sum_{k=0}^{\infty} \left[\tilde{x}_i^T(k)\tilde{Q}_i\tilde{x}_i(k) + u_i^T(k)R_iu_i(k)\right] \tag{5}$$

The corresponding discrete-time algebraic Riccati equation is then given by:

$$\tilde{P}_i = \tilde{A}_i^T\tilde{P}_i\tilde{A}_i - \tilde{A}_i^T\tilde{P}_i\tilde{B}_i\left(R_i + \tilde{B}_i^T\tilde{P}_i\tilde{B}_i\right)^{-1}\tilde{B}_i^T\tilde{P}_i\tilde{A}_i + \tilde{Q}_i \tag{6}$$

From this, the state feedback gain matrix is obtained as:

$$K_i = (R_i + \tilde{B}_i^T\tilde{P}_i\tilde{B}_i)^{-1}\tilde{B}_i^T\tilde{P}_i\tilde{A}_i \tag{7}$$

Accordingly, the control signal is computed as:

$$u_i(k) = -K_i\tilde{x}_i(k) = -K_i^x x_i(k) - K_i^z z_i(k) \tag{8}$$

We can adopt a more general approach to facilitate computation by solving the Riccati equation only once. The main idea is to aggregate the variables and matrices of all subsystems into a single large-scale system. This enables the controller design to require only one solution of the Riccati equation. Note that the aggregation of state variables affects only the controller design phase; the resulting controller matrix remains a block structure that contains the subcontrollers, identical to the previous approach.

According to the study in [14], we aggregate the state variables, control signals, and reference signals as follows:

$$X(k) = \begin{bmatrix} \tilde{x}_1(k) \\ \vdots \\ \tilde{x}_a(k) \end{bmatrix} \in \mathbb{R}^{a(n+p)},\ U(k) = \begin{bmatrix} u_1(k) \\ \vdots \\ u_a(k) \end{bmatrix} \in \mathbb{R}^{am} \tag{9}$$

$$R(k) = \begin{bmatrix} r_1(k) \\ \vdots \\ r_a(k) \end{bmatrix} \in \mathbb{R}^{ap}$$

From the transformations in (4) and (9), we obtain the following aggregated system model:

$$X(k+1) = \begin{bmatrix} \tilde{A}_1 & 0 & \cdots & 0 \\ 0 & \tilde{A}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{A}_a \end{bmatrix} X(k) + \begin{bmatrix} \tilde{B}_1 & 0 & \cdots & 0 \\ 0 & \tilde{B}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{B}_a \end{bmatrix} U(k) + \begin{bmatrix} \tilde{E} & 0 & \cdots & 0 \\ 0 & \tilde{E} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{E} \end{bmatrix} R(k) \tag{10}$$

Define: $N = I_a \otimes \tilde{A}_i$, $M = I_a \otimes \tilde{B}_i$, $Q^+ = I_a \otimes \tilde{Q}_i$, $R^+ = I_a \otimes R_i$, $E = I_a \otimes \tilde{E}$. Substituting into the model (10), we obtain the following discrete-time linear system:

$$X(k+1) = NX(k) + MU(k) + ER(k) \tag{11}$$

Analogous to (5), the cost function is formulated as:

$$J = \sum_{k=0}^{\infty} \left[X^T(k)Q^+X(k) + U^T(k)R^+U(k)\right] \tag{12}$$

We then solve a generalized Riccati equation that simultaneously accounts for all a equations in (6) through a single computation:

$$P^+ = N^TP^+N - N^TP^+M\left(R^+ + M^TP^+M\right)^{-1}M^TP^+N + Q^+ \tag{13}$$

Consequently, the aggregated feedback gain matrix and control signal are given by:

$$K^+ = (R^+ + M^TP^+M)^{-1}M^TP^+N \tag{14}$$

$$U(k) = -K^+X(k) \tag{15}$$

The controller for the i-th subsystem can be extracted from the aggregated gain matrix K^+ as:

$$K_i = K^+_{[(i-1)m+1:im,\ :]} \in \mathbb{R}^{m\times(n+p)}, \quad \forall i = 1,\ldots,a \tag{16}$$

3. Centralized Controller Design with Shared Control Input

Consider the system of the form (1) but with identical subsystems, and with the optimal matrices Q and R given as:

$$\begin{cases} x_i(k+1) = A_ix_i(k) + B_iu_i(k) \\ y_i(k) = C_ix_i(k) \end{cases} \tag{17}$$

Assume that all the above subsystems are controllable and observable. We transform model (17) into the reference-tracking control model (4), yielding:

$$\tilde{x}_i(k+1) = \tilde{A}_i\tilde{x}_i(k) + \tilde{B}_i u_i(k) + \tilde{E} r_i(k) \tag{18}$$

In [12] it is proven that the LTI system $(\tilde{A}_i, \tilde{B}_i)$ is stabilizable.

Next, we apply the design method as in model (10), but eliminate some variables and aggregate the matrices $\tilde{A}$ and $\tilde{E}$ to obtain:

$$X(k+1) = \begin{bmatrix} \tilde{x}_1(k+1) \\ \tilde{x}_2(k+1) \\ \vdots \\ \tilde{x}_a(k+1) \end{bmatrix} = \begin{bmatrix} \tilde{A}_1 & 0 & \cdots & 0 \\ 0 & \tilde{A}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{A}_a \end{bmatrix} \begin{bmatrix} \tilde{x}_1(k) \\ \tilde{x}_2(k) \\ \vdots \\ \tilde{x}_a(k) \end{bmatrix} + \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \\ \vdots \\ \tilde{B}_a \end{bmatrix} u(k) + \begin{bmatrix} \tilde{E} & 0 & \cdots & 0 \\ 0 & \tilde{E} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{E} \end{bmatrix} \begin{bmatrix} \tilde{r}_1(k) \\ \tilde{r}_2(k) \\ \vdots \\ \tilde{r}_a(k) \end{bmatrix} \tag{19}$$

Lemma: Looking at (18), the deviations of the subsystems are $e_1(k), e_2(k), \ldots, e_a(k)$. The goal of the control is to drive these deviations to zero. To achieve this, we need to compute a distinct control signals $u_1(k), u_2(k), \ldots, u_a(k)$ via controllers K_i based on the states $\tilde{x}_i(k)$.

Assume $A_i = A$, $B_i = B$, $Q_i = Q$, $R_i = R$, $C_i = C$, i.e., all subsystems are identical, and the reference signal $r(k)$ is common to every subsystem. We can draw the following conclusions: All controllers K_i designed in (7) coincide and equal K, because the Riccati equation (6) are identical. At the equilibrium $\tilde{x}^* = \begin{bmatrix} x_{\text{const}} & z_{\text{const}} \end{bmatrix}$, the control signal given by formula (8) for all a subsystems is the same, namely $u^*(k) = K\tilde{x}^*$. This raises an important question: Is it possible to use a single control signal to regulate all subsystems simultaneously?

Theorem: It is possible to use a single control signal to regulate the system composed of a identical subsystems sharing a common reference signal.

Consider the system of the form (17) under the assumptions stated in the Lemma:

$$\begin{cases} x_i(k+1) = A x_i(k) + B u_i(k) \\ y_i(k) = C x_i(k) \end{cases} \tag{20}$$

Assume all subsystems are controllable and observable, i.e.:

$$\begin{aligned} &\operatorname{rank}\begin{bmatrix} B \; AB \; A^2B \; \cdots \; A^{n-1}B \end{bmatrix} = n, \\ &\operatorname{rank}\begin{bmatrix} C^T \; (CA)^T \; (CA^2)^T \; \cdots \; (CA^{n-1})^T \end{bmatrix} = n \end{aligned} \tag{21}$$

We transform model (20) into the reference-tracking form (4) as follows:

$$\tilde{x}_i(k+1) = \tilde{A}\tilde{x}_i(k) + \tilde{B} u_i(k) + \tilde{E} r_i(k) \tag{22}$$

In [12] it is shown that the LTI pair $(\tilde{A}, \tilde{B})$ is stabilizable.

Next, applying the design of model (10) but eliminating some variables and aggregating the matrices, we get:

$$\begin{aligned} X(k+1) &= \begin{bmatrix} \tilde{x}_1(k+1) \\ \tilde{x}_2(k+1) \\ \vdots \\ \tilde{x}_a(k+1) \end{bmatrix} = \begin{bmatrix} \tilde{A} & 0 & \cdots & 0 \\ 0 & \tilde{A} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{A} \end{bmatrix} \begin{bmatrix} \tilde{x}_1(k) \\ \tilde{x}_2(k) \\ \vdots \\ \tilde{x}_a(k) \end{bmatrix} \\ &\quad + \begin{bmatrix} \tilde{B} \\ \tilde{B} \\ \vdots \\ \tilde{B} \end{bmatrix} u(k) + \begin{bmatrix} \tilde{E} & 0 & \cdots & 0 \\ 0 & \tilde{E} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{E} \end{bmatrix} \begin{bmatrix} \tilde{r}_1(k) \\ \tilde{r}_2(k) \\ \vdots \\ \tilde{r}_a(k) \end{bmatrix} \\ &= \Psi X(k) + \Lambda u(k) + \Phi R(k) \end{aligned} \tag{23}$$

In this new model, all subsystems share the same control signal $u(k)$ and common reference, while their states remain independent.

Define the cost functional that evaluates the overall system performance under a centralized control input.

$$J = \sum_{k=0}^{\infty} \left[X(k)^T Q^\times X(k) + u(k)^T R^\times u(k) \right] \tag{24}$$

The discrete Riccati equation is

$$\begin{aligned} &\Psi^T P^\times \Psi - P^\times - \Psi^T P^\times \Lambda (\Lambda^T P^\times \Lambda + R^\times)^{-1} \\ &\times \Lambda^T P^\times \Psi + Q^\times = 0 \end{aligned} \tag{25}$$

Let $P^\times = I_a \otimes \hat{P}$, $\quad \Psi = I_a \otimes \tilde{A}$, $\quad \Lambda = \mathbf{1}_a \otimes \tilde{B}$, $Q^\times = I_a \otimes \tilde{Q}$, $\quad R^\times = R$.

Then one finds

$$\begin{cases} \Psi^T P^\times \Psi &= (I_a \otimes \tilde{A})^T (I_a \otimes \hat{P})(I_a \otimes \tilde{A}) = I_a \otimes \tilde{A}^T \hat{P} \tilde{A} \\ \Psi^T P^\times \Lambda &= (I_a \otimes \tilde{A})^T (I_a \otimes \hat{P})(1_a \otimes \tilde{B}) = 1_a \otimes \tilde{A}^T \hat{P} \tilde{B} \\ \Lambda^T P^\times \Lambda &= (1_a \otimes \tilde{B})^T (I_a \otimes \hat{P})(1_a \otimes \tilde{B}) = a\tilde{B}^T \hat{P} \tilde{B} \\ \Lambda^T P^\times \Psi &= (1_a \otimes \tilde{B})^T (I_a \otimes \hat{P})(I_a \otimes \tilde{A}) = 1_a^T \otimes \tilde{B}^T \hat{P} \tilde{A} \end{cases} \tag{26}$$

Substituting (26) into (25) and grouping via Kronecker products yields

$$I_a \otimes \left[\tilde{A}^T \hat{P} \tilde{A} - \hat{P} - a\tilde{A}^T \hat{P} \tilde{B} (a\tilde{B}^T \hat{P} \tilde{B} + R)^{-1} \tilde{B}^T \hat{P} \tilde{A} + \tilde{Q} \right] = 0 \tag{27}$$

so, the reduced Riccati equation is

$$\tilde{A}^T\hat{P}\tilde{A}-\hat{P}-a\tilde{A}^T\hat{P}\tilde{B}(a\tilde{B}^T\hat{P}\tilde{B}+R)^{-1}\tilde{B}^T\hat{P}\tilde{A}+\tilde{Q}=0 \quad (28)$$

Multiplying both sides by a gives

$$\begin{aligned}&\tilde{A}^T(a\hat{P})\tilde{A}-(a\hat{P})-\tilde{A}^T(a\hat{P})\tilde{B}[\tilde{B}^T(a\hat{P})\tilde{B}+R]^{-1}\\&\times\tilde{B}^T(a\hat{P})\tilde{A}+a\tilde{Q}=0\end{aligned} \quad (29)$$

which matches the form of (6). Since $\tilde{Q}\geq 0$ implies $a\tilde{Q}\geq 0$ and $(\tilde{A},\tilde{B})$ is stabilizable, for $R>0$ there exists a unique positive-definite solution $a\hat{P}$. Thus $P^{\times}>0$ solves (25), proving stability.

The optimal gain is

$$\begin{aligned}K^{\times}&=(\Lambda^T P^{\times}\Lambda+R^{\times})^{-1}\Lambda^T P^{\times}\Psi\\&=\left(a\tilde{B}^T\hat{P}\tilde{B}+R\right)^{-1}\left(\mathbf{1}_a^T\otimes\tilde{B}^T\hat{P}\tilde{A}\right)\end{aligned} \quad (30)$$

and the control law is

$$u(k)=-K^{\times}X(k)=-K(\tilde{x}_1+\tilde{x}_2+\ldots+\tilde{x}_a) \quad (31)$$

Hence, a single control signal $u(k)$ suffices to regulate all a identical subsystems with the shared reference.

4. Simulation and Results

Note: the symbols a_1,a_2,a_3,a_4 used in the four-tank parameters denote orifice areas and are unrelated to the symbol a representing the number of subsystems in the control formulation.

To evaluate the performance of the centralized controller using a shared control input, we conducted a series of simulation experiments on various configurations of the extended four-tank system. Initially, the system was initialized under different conditions, including random deviations in water levels in one or more tank clusters, as well as step disturbances or stochastic noise affecting the reference signal. The sampling time T_s was set to 0.1 seconds, which is consistent with the operational characteristics of the actual four-tank experimental setup.

Remark: Because this is an experimental-type model, disturbances naturally occur due to the cross-coupling between tanks. For instance, variations in the water level of Tank 3 influence the dynamics of Tank 1 through the connecting channels and the shared control action of Pump 2. This structural interaction is treated as a form of disturbance in the simulation to better reflect real experimental behavior.

In the four-tank system shown in Fig. 1, each tank is characterized by its water level h_i ($i = 1,\ldots,4$) and there are two pumps with control signals v_j ($j = 1,2$). The physical parameters include the tank cross-sectional area $A_i = 730\times 10^{-4}\,\mathrm{m}^2$, outlet orifice areas $(a_1,a_2,a_3,a_4) = (2.10,2.14,2.20,2.30)\times$

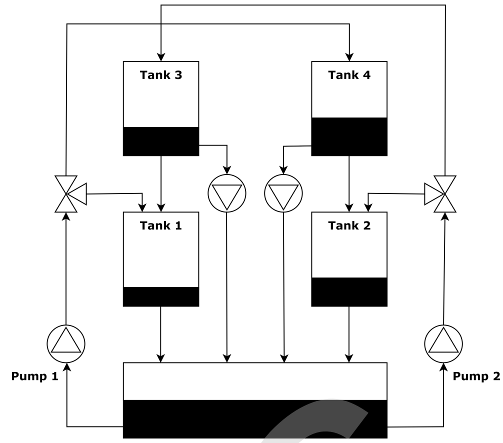

Fig. 1. The four-tank experimental system model

$10^{-4}\,\mathrm{m}^2$, flow distribution coefficients $\gamma_1 = 0.30$, $\gamma_2 = 0.35$, pump gain constants $k_j = (7.45,7.30)\times 10^{-4}\,\mathrm{m}^3/(\mathrm{s}\cdot\%)$, pump time constants $\tau = (2.0,2.1)\,\mathrm{s}$, and gravitational acceleration $g = 981\times 10^{-4}\,\mathrm{m/s}^2$. The control objective is to regulate the water levels of the two lower tanks h_1 and h_2 to track the reference signals, while minimizing energy consumption and limiting oscillations across the entire system. According to [9], the dynamic behavior of the system is described by the following set of differential equations:

$$\begin{aligned}\frac{\mathrm{d}h_1}{\mathrm{d}t}&=-\frac{a_1}{A_1}\sqrt{2gh_1}+\frac{a_3}{A_1}\sqrt{2gh_3}+\frac{\gamma_1 k_1}{A_1}v_1+\frac{d}{A_1}\\\frac{\mathrm{d}h_2}{\mathrm{d}t}&=-\frac{a_2}{A_2}\sqrt{2gh_2}+\frac{a_4}{A_2}\sqrt{2gh_4}+\frac{\gamma_2 k_2}{A_2}v_2-\frac{d}{A_2}\\\frac{\mathrm{d}h_3}{\mathrm{d}t}&=-\frac{a_3}{A_3}\sqrt{2gh_3}+\frac{(1-\gamma_2)k_2}{A_3}v_2\\\frac{\mathrm{d}h_4}{\mathrm{d}t}&=-\frac{a_4}{A_4}\sqrt{2gh_4}+\frac{(1-\gamma_1)k_1}{A_4}v_1\\\frac{\mathrm{d}v_1}{\mathrm{d}t}&=-\frac{v_1}{\tau_1}+\frac{1}{\tau_1}u_1\\\frac{\mathrm{d}v_2}{\mathrm{d}t}&=-\frac{v_2}{\tau_2}+\frac{1}{\tau_2}u_2\end{aligned} \quad (32)$$

In the four-tank system model, we define the state vector as $x=[h_1,\,h_3,\,v_2,\,v_1,\,h_2,\,h_4]^T$, where h_i denotes the water level in tank i, and u_1 and u_2 denote the voltage control signals applied to Pumps 1 and 2, while q_j represents the corresponding outlet flow rates of each pump. The control input is given by $u=[v_1,\,v_2]^T$, with v_j being the control signal (percentage of power) sent to pump j. The system is linearized and then discretized around the nominal operating point into the following state-space representation:

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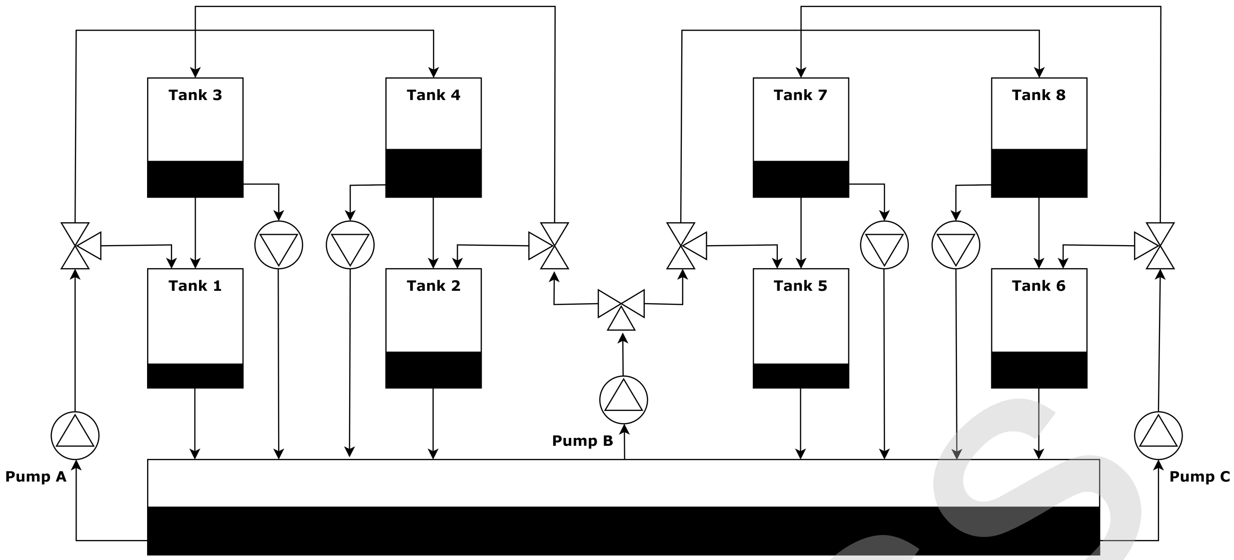

Fig. 2. Extended four-tank experimental system model with a shared control input

$$x(k+1) = \begin{bmatrix} 0.982 & 0.028 & 0 & 0.001 & 0 & 0 \\ 0 & 0.972 & 0.002 & 0 & 0 & 0 \\ 0 & 0 & 0.607 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.607 & 0 & 0 \\ 0 & 0 & 0.001 & 0 & 0.981 & 0.034 \\ 0 & 0 & 0 & 0.002 & 0 & 0.965 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 0 & 0.001 \\ 0 & 0.787 \\ 0.787 & 0 \\ 0 & 0 \\ 0.001 & 0 \end{bmatrix} u(k), \tag{33}$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} x(k) \tag{34}$$

The configuration of the extended four-tank experimental system is illustrated in Fig. 2. The setup has been expanded into two identical clusters that share a single control input. Each cluster consists of four interconnected tanks: Tanks 1–4 in Cluster A and Tanks 5–8 in Cluster C. Within this control architecture, a single control signal is delivered to Pump B, simultaneously affecting tanks in both clusters.

In particular, Tanks 3 and 5 from the two different clusters receive water from the same pump, implying that any variation in the control signal influences both tanks concurrently. This arrangement simplifies the hardware and reduces implementation cost; however, it introduces control challenges in coordinating the response of both clusters while still ensuring effective setpoint tracking and disturbance rejection.

Such a configuration exemplifies a practical issue encountered in large-scale systems, namely how to design an effective centralized controller under structural constraints such as shared actuators or control signals. The simulation results presented in the subsequent sections demonstrate that, under appropriate conditions, a single control input can still achieve satisfactory closed-loop performance for the entire system.

Fig. 3 through Fig. 8 present the simulation results used to evaluate and compare the performance of two distinct centralized control architectures applied to the extended four-tank experimental system. These architectures include (i) a conventional centralized control structure, where each subsystem cluster is regulated by an independent control signal, and (ii) a centralized control structure employing a shared control signal for multiple subsystem clusters. The evaluations focus on reference tracking capability, oscillation magnitude, system stability, and potential reductions in the number of required control inputs.

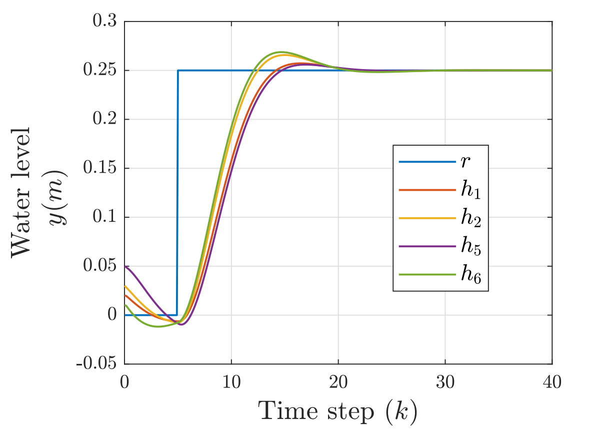

Fig. 3. Output signals under conventional centralized control design

As shown in Fig. 3, the output signals h_1, h_2, h_5, and h_6 converge rapidly toward the reference value $r = 0.25$ m within approximately 20–30 timesteps. Despite an initial overshoot of roughly 8–10%, the fast settling behavior demonstrates the effectiveness and stability of this control scheme.

The control signals u_1–u_4, depicted in Fig. 4, exhibit moderate amplitudes and quick stabilization following initial transients. This clearly indicates that each tank cluster can respond independently and maintain balanced control, thereby contributing to overall system stability.

Tracking errors, illustrated in Fig. 5, initially remain below 0.3 m and diminish swiftly to nearly zero within about 20 timesteps. The uniform responses across different clusters confirm that this conventional architecture efficiently distributes the control effort, ensuring even and stable tracking performance.

Under the shared control input, the output signals displayed in Fig. 6 show markedly greater oscillations compared to the conventional case. Tank h_5, in particular, experiences pronounced oscillations and requires approximately 250–300 timesteps to stabilize at the reference point, clearly illustrating the limitations of applying a single control signal across subsystems with differing initial conditions.

The control signals u_A, u_B, and u_C, presented in Fig. 7, exhibit stronger and more prolonged oscillations, especially in the shared control signal u_B. Nevertheless, this configuration substantially reduces the number of control variables and actuator elements (e.g., control valves), thereby simplifying hardware requirements and lowering implementation costs.

As demonstrated in Fig. 8, the tracking errors under the shared control approach are initially higher (up to 0.25 m) and require a substantially longer duration (more than 250 timesteps) to reach a steady-state near the reference. Nonetheless, the system ultimately achieves stability, albeit with reduced convergence performance.

As observed in Table 1, the centralized control design using a shared control input offers notable advantages in terms of hardware simplicity and reduced input requirements. However, these benefits come with clear compromises in dynamic performance, including slower response and more pronounced oscillations.

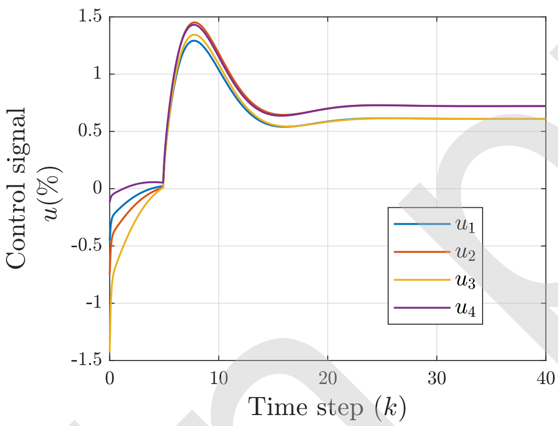

Fig. 4. Control signals under conventional centralized control design

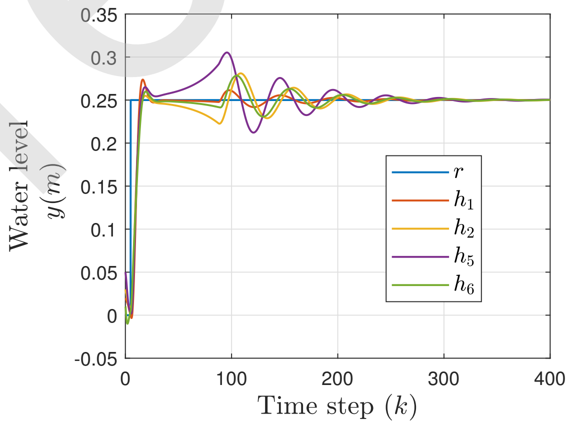

Fig. 6. Output signals under centralized control design with a shared control input

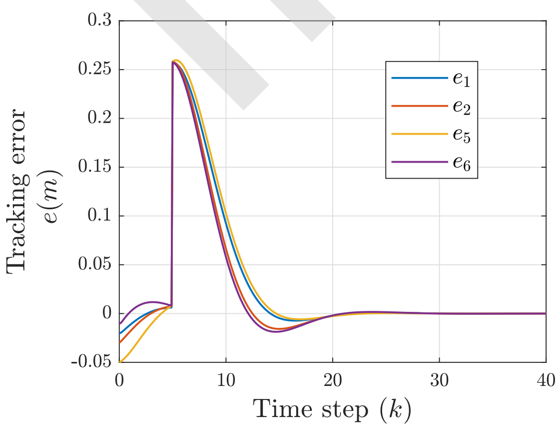

Fig. 5. Tracking errors under conventional centralized control design

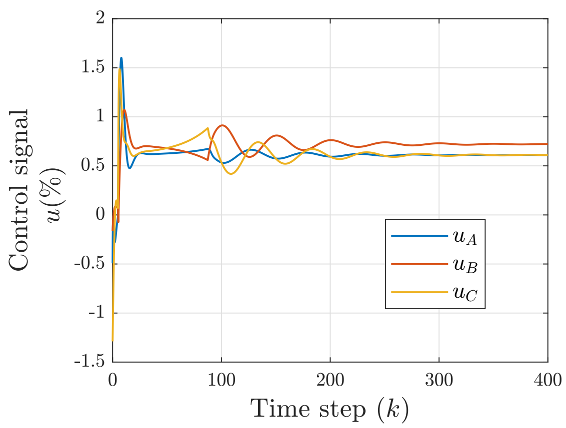

Fig. 7. Control signals under centralized control design with a shared control input

This trade-off suggests that such an approach is best suited for large-scale systems where cost efficiency and structural uniformity are prioritized over precision and fast settling behavior. From these simulation analyses, it can be concluded that the conventional centralized control structure demonstrates superior performance in accuracy, response speed, and stability at the cost of increased hardware complexity and associated expenses. Meanwhile, the shared control signal offers clear advantages in terms of significantly reducing the number of control inputs and hardware complexity, despite some trade-offs in convergence time and increased oscillations.

The proposed concept has so far only been verified through simulations using a mathematical model. To enable practical deployment, it is essential to consider real-world constraints such as minimum tank water levels and limits on actuator power. In practice, such constraints are often handled effectively within Model Predictive Control (MPC) frameworks. Recent developments have shown that centralized control architectures, when structured hierarchically or with appropriate decomposition, can be successfully applied in large-scale or real-time systems. By organizing subsystems into coordinated groups and using multi-level control strategies, it is possible to achieve performance close to optimal while keeping the computational requirements within practical limits. These findings indicate that the proposed shared-signal centralized control method could be realistically implemented on embedded systems or physical platforms with acceptable complexity.

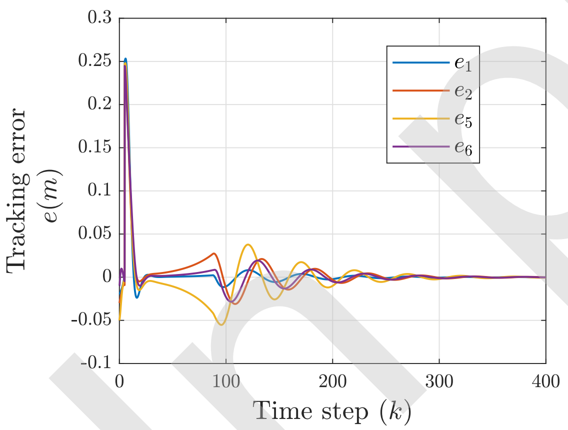

Fig. 8. Tracking errors under centralized control design with a shared control input

5. Conclusion

This paper has presented a centralized optimal control framework that utilizes a shared control signal to regulate multiple structurally identical subsystems under a common reference. Through theoretical formulation and simulation studies on a multi four-tank benchmark system, the proposed approach has demonstrated the potential to reduce the number of control signals and actuators, thereby simplifying the overall control structure and lowering implementation costs in principle. Although the shared-input configuration exhibits moderate performance degradation, such as slower convergence and larger oscillations, it still ensures closed-loop stability and acceptable tracking performance within the tested scenarios. These results indicate that the proposed method is feasible for homogeneous large-scale systems where cost efficiency and structural uniformity are prioritized over high dynamic performance.

It should be noted that the current validation is limited to simulation-based analysis, and further investigation on experimental platforms is required to assess practical implementation aspects, including hardware nonlinearity and actuator saturation. Future work will focus on extending the framework to slightly heterogeneous subsystems, incorporating actuator and communication constraints, and integrating predictive or fault-tolerant control strategies. Overall, the presented study provides an initial yet insightful step toward scalable and resource-efficient centralized control architectures for large-scale and cooperative system applications.

Table 1. Comparison of control performance between conventional centralized control design and centralized control design using a shared control input

Metric	**Conventional Centralized Control**	**Shared-Input Centralized Control**
Settling Time	20 – 30 steps	250 – 300 steps
Max Overshoot	0.020 – 0.025 m	0.025 – 0.060 m
Control Signals Used	4 separate signals	2 separate signals and 1 shared signal
Tracking Accuracy	High	Acceptable (some lag)
Oscillation Level	Low	High
Control Design Complexity	High (individual tuning)	Low (shared gain)
Hardware Cost	High	Low
Scalability	Limited	Good for large systems

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