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Swing-Up and Position Control of Inverted Pendulum - Cart Systems Using Optimized Fuzzy Controller

Hai-Le Bui, Thi-Thoa Mac*
School of Mechanical Engineering, Hanoi University of Science and Technology, Ha Noi, Vietnam
**Corresponding author email: thoa.macthi@hust.edu.vn*

Abstract

The study proposes a simple approach to design optimally a fuzzy controller in swing-up and position control for inverted pendulum-cart systems. First, the sub-fuzzy controllers to control the pendulum's swing up and the cart's position are designed separately. Each controller includes two state variables to calculate the component control forces. The combination of component control forces determines the final control force through a weight computed from a simple scheme. Parameters of sub-fuzzy controllers and those to determine the weight are optimized to minimize the system's equilibration time. The simulation results show that the proposed controller is simple to set up and optimize, has high control efficiency, is adaptable to the system's state, and is stable and robust for the system's different initial conditions and configurations. When using the proposed controller, the stabilization time of the system is reduced by 14.5%, the maximum control force is reduced by 32.6%, and the pendulum length is increased by 50% compared to fuzzy controllers in the published studies. The approach of the present work can be applied to control various underactuated systems as well as in the motion control of mobile robot models.

Keywords: Inverted pendulum-cart, swing-up and position control, fuzzy control, sub-controllers, design optimization.

1. Introduction

Pendulum models have many applications in industry in general and in robotics in particular. These models can be inverted pendulum-cart, rotary-inverted pendulum, gantry crane, two-wheeled inverted pendulum, acrobot, pendubot, and so on. The above models are usually nonlinear and underactuated systems. Therefore, they are highly challenging research candidates in the field of process control [1]. Many modern and intelligent control algorithms have been applied to control the balance for pendulum systems, such as Lyapunov-based [2], asymptotic stabilization approach [3], proportional–integral–derivative (PID) [4], sliding mode [5], adaptive [6], robustness [7], the linear quadratic regulator (LQR) [8], fuzzy and hedge-algebras-based [9], and neural network-based [10] controllers.

The inverted pendulum models are also studied objects to validate the performance of controllers because of their complexity and nonlinearity. Controllers based on fuzzy set theory have many advantages, such as simplicity and flexibility in setup. They can be applied to complex problems where the modeling of the controlled object is difficult [11]. Hence, the control based on fuzzy set theory is one of the most popular methods for inverted pendulum models. The cart-pendulum-seesaw system's balancing and swing-up control problems were covered in [12] using the decomposed fuzzy coordination control. The fuzzy control problem was studied in [13] for the third-order inverted pendulum-cart model, which used three state variables, including the deflection angle and angular velocity of the pendulum and the cart's position, to calculate the driving force. The fuzzy controller to control the inverted pendulum-car system in [14] used the Takagi–Sugeno (TS) model with four state variables and sixteen control rules. The pendulum in the studies [15] is used as an axisymmetric actuator to control the motion of a spherical robot. The double inverted pendulum in [16] has been linearized based on the fuzzy state feedback technique to establish a fuzzy robustness controller for the stable control problem of the system. The type-2 fuzzy logic controller was optimized in [17] for an inverted pendulum-cart model based on the rule bases' continuity, monotonicity, and smoothness. Two TS fuzzy controllers were used in [18] to control the roll and pitch motions of an omnidirectional inverted pendulum. The swing-up control problem using the fuzzy controller for a twin-arm inverted pendulum cart was presented in [19]. The inverted pendulum-cart in [20] was controlled by type-2 fuzzy controllers with an opposition-based spiral dynamic algorithm. Two fuzzy systems with input as state variables of an inverted pendulum-cart system and output as control gains for a PID controller were proposed in [21] to swing up stabilization for the model. The optimal fuzzy fractional-order adaptive robust controller in motion control of an inverted pendulum model was proposed in [22].

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The fuzzy controllers in the above publications are often quite complex. They may contain multiple control rules, which are decomposed into intermediate controllers with complex inference to determine the final control action. Furthermore, these controllers often require advanced control techniques such as type-2 fuzzy logic or the Takagi-Sugeno model. The complexity is further increased by the need to integrate other control algorithms, such as PID, LQR, fractional-order techniques, optimal state feedback control, sliding mode control, feedback linearization, or opposition-based spiral dynamic algorithms. Sometimes, a switching algorithm is needed to coordinate the actions between these controllers, depending on the system state. Therefore, the design and optimization of these fuzzy controllers becomes quite complicated. This poses a necessary requirement: to develop a simple, independently operating fuzzy controller to solve the motion control problem for general underactuated systems, especially the position and swing-up motion control of inverted pendulum systems.

This study presents a simple approach to design and optimize a Mamdani-type fuzzy controller. The controller aims to simultaneously control the swing motion and position of an inverted pendulum-cart model.

The design consists of two sub-controllers that operate separately to maintain: (1) the swing motion of the pendulum and (2) the position of the cart. The overall driving force is determined by combining the outputs of these two sub-controllers through a weight calculated based on the current states of the pendulum and cart.

The optimization problem is performed based on important design variables, including: (1) parameters of membership functions, (2) reference ranges of variables, and (3) parameters used to determine the mentioned combination weight. The simulation results have demonstrated that the performance of the proposed fuzzy controller is superior to that of the controllers in previous publications. This is the outstanding contribution and core value of this study.

The content of the article consists of five sections. After this introduction, the investigation model is presented in Section 2. Section 3 offers the design steps of the fuzzy controller. The numerical simulation results are given in Section 4, and the conclusions are written in Section 5.

2. Investigation Model

Consider the inverted pendulum-cart model, as shown in Fig. 1. The masses of the pendulum and the cart are m_p and m_c, respectively. The pendulum's length is l_p, and l_c = OC = l_p /2.

Ignoring friction, let g be the gravitational acceleration; the equation of state for the system is as follows [23]:

$$\begin{aligned}
\ddot{\theta} &= \frac{\left(m_c + m_p\right) g \sin\theta - \left(u + m_p l_c \dot{\theta}^2 \sin\theta\right)\cos\theta}{\frac{4}{3}\left(m_c + m_p\right) l_c - m_p l_c \cos^2\theta} \\
\ddot{x} &= \frac{\frac{4}{3}\left(u + m_p l_c \dot{\theta}^2 \sin\theta\right) - m_p g \sin\theta\cos\theta}{\frac{4}{3}\left(m_c + m_p\right) - m_p \cos^2\theta}
\end{aligned} \tag{1}$$

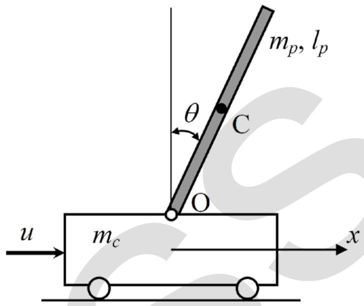

Fig. 1. Investigation model.

Here θ, $\dot{\theta}$, and $\ddot{\theta}$ are the pendulum's deflection angle, angular velocity, and angular acceleration, respectively; and x, $\dot{x}$, and $\ddot{x}$ are the cart's position, velocity, and acceleration, respectively. The control objective of the system is to bring the pendulum to the vertical equilibrium position (θ and $\dot{\theta}$ = 0) and the cart to its initial position (x and $\dot{x}$ = 0) from the non-zero initial conditions of the state variables (θ, $\dot{\theta}$, x, and $\dot{x}$). The control force u is calculated from the proposed fuzzy controller in Section 3. Equations of the state (1) will be solved by the 4th-order Runge-Kutta method [24] in the simulations of this study. The above four state variables (θ, $\dot{\theta}$, x, and $\dot{x}$) are both the initial conditions to solve (1) and inputs of the proposed fuzzy controller.

3. Fuzzy Control Design

It can be seen that controllers based on fuzzy set theory are popular for mechanical underactuated systems in general and inverted pendulum systems in particular. One of the reasons for their popularity is that these controllers are mathematically simple, which makes them advantageous when applied to complex and nonlinear systems. In addition, building a fuzzy control rule system for inverted pendulum systems is not difficult based on the experts' experience and observations. Therefore, the fuzzy controller is selected in this study. This section presents the steps to set up and optimize it to control the balance for the inverted pendulum-cart system.

The system's control diagram is presented in Fig. 2, in which FC1 and FC2 are the sub-fuzzy controllers to control the deflection angle of the pendulum and the position of the vehicle, respectively. The state and control variables of FC1 are (θ, $\dot{\theta}$) and u_1, and of FC2 are (x, $\dot{x}$) and u_2. Determining the weight w and the overall control force u will be explained below. The control force u will change the system's state (θ, $\dot{\theta}$, x, and $\dot{x}$). The system is stable when these variables reach zero, and the control process ends.

The fuzzy controllers FC1 and FC2 workflow is plotted in Fig. 3. These controls include the Fuzzification, Rule Base, Inference, and De-Fuzzification function blocks. The overall control force u is a function that depends on u_1, u_2, θ, and w.

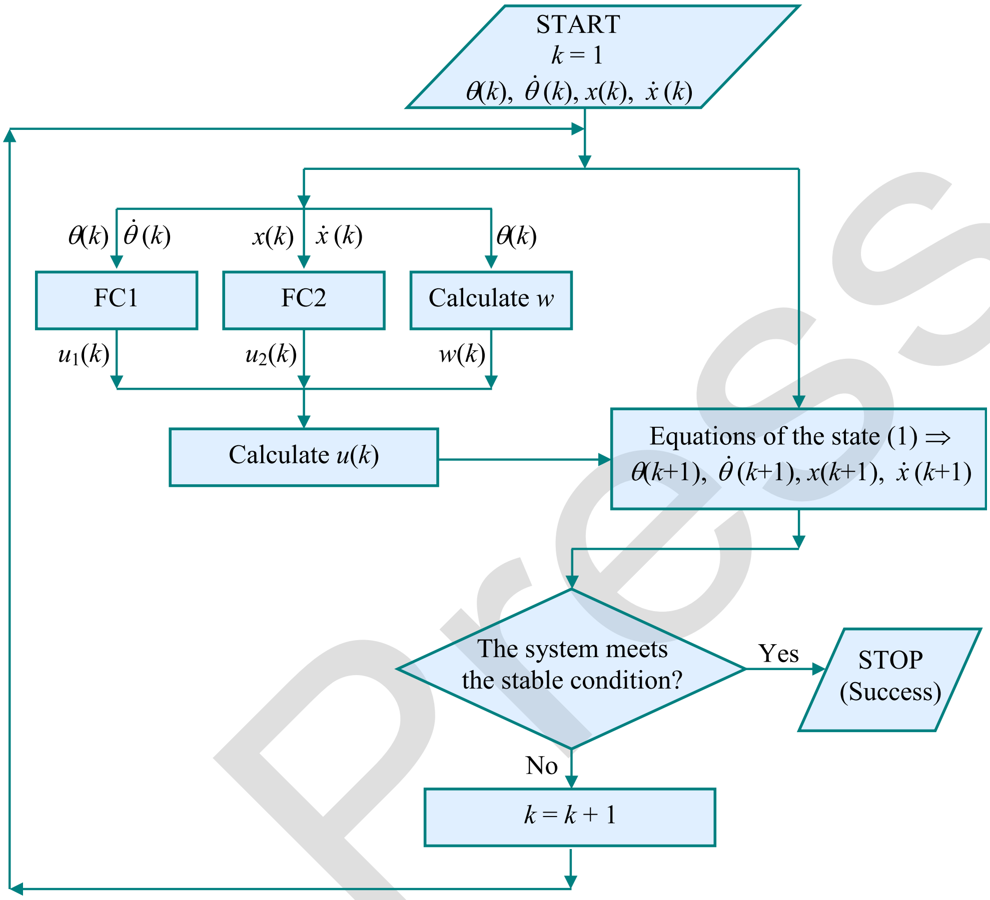

Fig. 2. System's control diagram

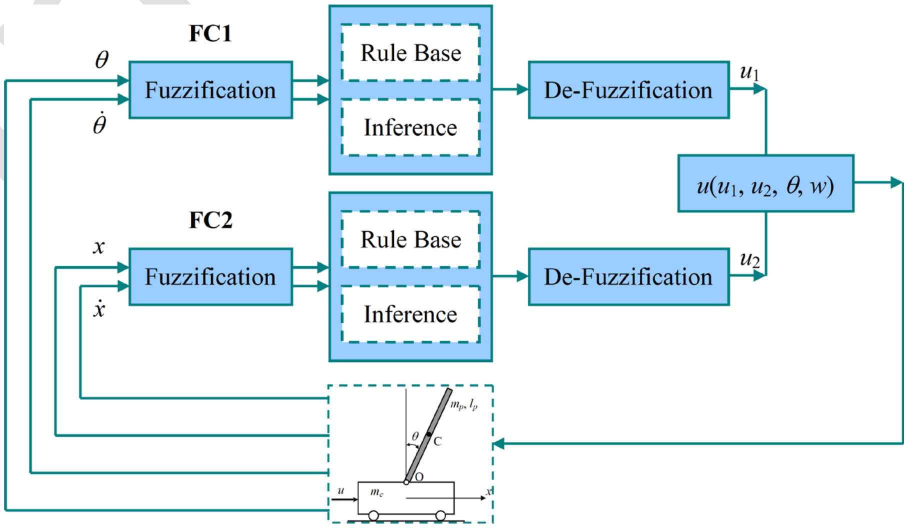

Fig. 3. Workflow of fuzzy controllers.

Hence, it can be seen from Fig. 2 and Fig. 3 that the calculated (or measured) values of the four state variables of the system (θ, $\dot{\theta}$, x, and $\dot{x}$) are used to calculate the control force u in the i^{th} loop through the proposed controller. These four state variables and the control force u are also the initial conditions and parameters for solving (1) by the 4th-order Runge-Kutta method. As a result, the values of the four state variables in the $(i+1)^{th}$ loop are determined. Subsequent iterations are performed until the system reaches the equilibrium state.

Assume that the variables have their reference interval and linguistic values as arranged in Table 1, where N, Z, P, and V are the notations of Negative, Zero, Positive, and Very, respectively.

Symmetrical fuzzification diagram for state variables (θ, $\dot{\theta}$, x, and $\dot{x}$) and control variables u_1 and u_2 using triangular membership functions are plotted in Fig. 4 and Fig. 5. The control rule bases of FC1 and FC2 are arranged in Tables 2 and 3. These control rules are established based on observing the system's behavior when affecting the control force. For example, when the deflection angle and angular velocity of the pendulum are negative and large, the driving force u needs to be very large, and its direction of action is to the left. When the deflection angle of the pendulum is negative, and its angular velocity is positive, the driving force u only needs a very small value around 0. For the cart, when its position and velocity are negative and large, the driving force u needs to be very large, and its direction is to the right. When the cart's position is positive, and its velocity is approximately zero, the control force u needs to be positive. By similar analysis for each state of the pendulum and cart, the control rule bases for controllers FC1 and FC2 are selected, as shown in Tables 2 and 3.

The FC1 and FC2 controllers use Mamdani and centroid methods for their Inference and Defuzzification steps. It can be seen that the number of linguistic values used for variables is minimal. In this sense, the number of control rules of FC1 and FC2 is also minimal.

It can be seen that the controllers FC1 and FC2 operate independently based on the states of the pendulum and the cart, respectively. However, this model is underactuated; only one actuator generates the control force u. Therefore, after obtaining the intermediate control forces (u_1 and u_2) from the controllers FC1 and FC2, the calculation of the overall control force u based on u_1, u_2, and the weight $w \in [0, 1]$ is proposed as follows:

$$u = wu_1 + (1-w)u_2 \tag{2}$$

in which the weight w is determined based on the importance of the state of the inverted pendulum and the cart.

Table 1. The variables' reference interval and linguistic values

Variable	Reference interval	Linguistic values
θ	$[-\theta^*, \theta^*]$	N, Z, P
$\dot{\theta}$	$[-\theta_d, \theta_d]$	N, Z, P
u_1	$[-u_{1*}, u_{1*}]$	VN, N, Z, P, VP
x	$[-x^*, x^*]$	N, Z, P
$\dot{x}$	$[-x_d, x_d]$	N, Z, P
u_2	$[-u_{2*}, u_{2*}]$	VN, N, Z, P, VP

Table 2. Control rule base of FC1

$\dot{\theta}$	N	Z	P
θ			
N	N	Z	P
Z	Z	P	VP
P	VN	N	Z

Table 3. Control rule base of FC2

$\dot{x}$	N	Z	P
x			
N	VP	P	Z
Z	P	Z	N
P	Z	N	VN

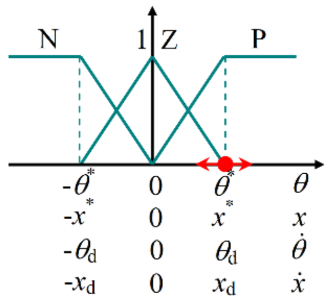

Fig. 4. Fuzzification of θ, x, $\dot{\theta}$, and $\dot{x}$

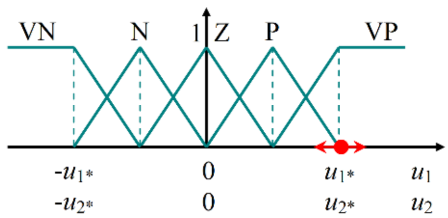

Fig. 5. Fuzzification of u_1 and u_2

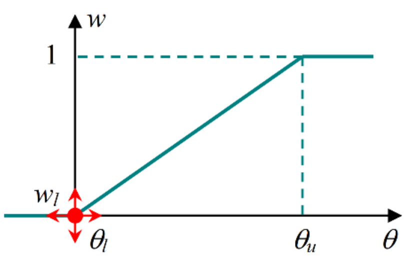

Fig. 6. The weight w versus θ

In this study, the proposed scheme to determine the weight w is plotted in Fig. 6. It can be seen that when the deflection angle θ is large (θ_u), the system is in a dangerous state. The overall control force u need to prioritize the swing-up control of the pendulum, and hence w can be up to 1. Conversely, when the deflection angle θ is small (θ_l), w can be approximately a given value w_l, the pendulum and the cart can return to their equilibrium positions simultaneously.

It should be noted that the fuzzy controller for controlling the inverted pendulum-cart model in Bui *et. al.* [25] (denoted by FC_B) consists of four intermediate controllers (for each state variable) with four combined weights. These weights are determined by the trial-and-error method. Hence, the parameters of this controller may not allow the system to operate most effectively.

In this study, the optimization problem of controllers FC1 and FC2 and the scheme to determine w are performed to improve the system's performance. The objective of this problem is to minimize the system's equilibrium time (t_s) from different initial conditions as follows:

$$t_s \rightarrow \min \tag{3}$$

It can be seen that the adjustable parameters of the controllers include the reference range of variables, as shown by the circle dots on Fig. 4 and Fig. 5. Moreover, the parameters θ_l, θ_u, and w_l to determine the weight w can also be optimized, as presented in Fig. 6. Therefore, the number of design variables of the optimization problem is 9 (θ^*, θ_d, u_{1*}, x^*, x_d, u_{2*}, θ_l, θ_u, and w_l).

This study uses the balancing composite motion optimization (BCMO) [26] algorithm to perform the optimization problem. This algorithm is a meta-heuristic, swarm-based, and recently published optimization technique. The idea of BCMO is to balance individuals' exploration and exploitation motions in the search for a better fitness function value at each iteration. BCMO has advantages such as no need for algorithm parameters, fast convergence, and high optimal performance. The effectiveness of BCMO has been verified through many simulations in different studies. The diagram of the optimal problem is represented in Fig. 7, in which maxGen is the number of generations or iterations in the optimization process.

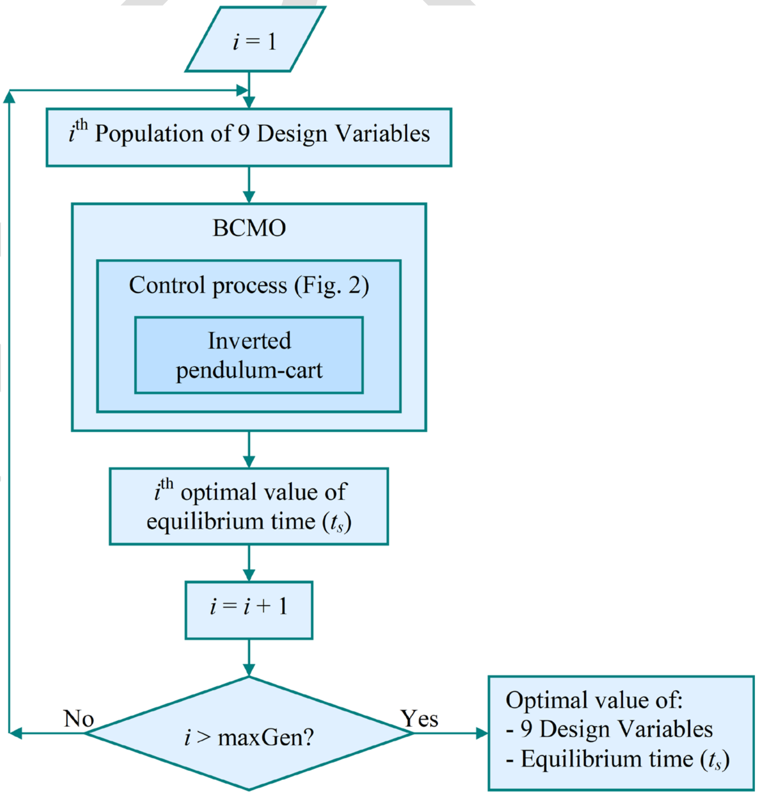

Fig. 7. The diagram of the optimal problem

4. Numerical Simulation

In this section, the numerical simulation results are presented. After performing the optimization problem to determine the suitable parameters for the fuzzy controllers FC1 and FC2 and the weighting scheme w, the numerical investigations include: (1) Comparison of the obtained results in this study with those in published articles; (2) Effect of the pendulum length on the system's equilibrium time t_s (s), and (3) Determine the maximum values of the pendulum's mass and length and the cart's mass with the proposed controller.

First, the optimization problem is performed using training data including (m_p, l_p, m_c) equal (0.1, 1, 1), (0.1, 0.2, 1), and (0.1, 2.2, 1) with the initial condition $\theta(0)$ equal 30° (the initial condition of the remaining state variables is 0). Here, the mass is in kg, and the length is in m. The allowable control force is u_{max} equal 50 N. The system is considered stable when [23]:

$$\theta \le 0.1^o;\quad \dot{\theta} \le 0.1^o/\text{s};\quad x \le 0.01\text{m};\quad \dot{x} \le 0.01\text{m/s} \tag{4}$$

The optimal values of the design variables are arranged in Table 4. From the results in Table 4, it can be seen that the optimization problem is necessary to determine the appropriate parameters of the controller. Because of the large number of design variables (9), it is not feasible to determine their values using experience or trial-and-error steps.

Second, several typical cases are simulated to verify the effectiveness of the proposed controller, as listed in Table 5. The simulation results of the fuzzy controller in Yi and Yubazaki [23] (denoted by FC_Y) and FC_B are also included for comparison. The results in Table 5 show that the controller proposed in this paper (FC) has about 14.5% and 6.5% faster stabilization time (t_s) and about 18.2% and 32.6% smaller maximum control force when compared with the controllers FC_Y and FC_B, respectively (according to the average value of four comparison cases). The time responses of the pendulum's deflection angle, cart's movement, and control force of the simulation cases in Table 5 are illustrated in Fig. 8. It can be seen from Fig. 8 that the vibration amplitude in the FC case is much smaller than that in the FC_B case. This phenomenon is because the slope of the control force of FC is larger than that of FC_B. However, this slope makes the cart's travel distance in the FC case larger than that in FC_B. In addition, investigations of the maximum initial deflection angle with the change of the pendulum's length are performed and presented in Fig. 9.

Table 4. The optimal values of the design variables

θ^*, °	θ_d, °/s	u_{1*}, N	x^*, m	x_d, m/s
106.08	326.79	538.62	15.79	8.20
u_{2*}, N	θ_l, °	θ_u, °	w_l	
269.43	5.86	70.00	0.48	

Table 5a. Typical simulation cases (t_s, s)

(m_p, l_p, m_c)	Initial condition	t_s, s		
		FC_Y	FC_B	FC
(0.1, 1, 1)	$\theta(0)$ = 30°	8.24	6.30	6.01
(0.1, 1, 1)	$x(0)$ = 2m	7.16	6.79	6.69
(0.1, 0.2, 1)	$\theta(0)$ = 30°	6.65	8.45	7.34
(0.5, 1, 1)	$\theta(0)$ = 30°	8.18	6.27	5.81
Mean value		7.56	6.95	6.46
Variation, %		0.00	-8.01	-14.49

Table 5b. Typical simulation cases (Max. of u, N)

(m_p, l_p, m_c)	Initial condition	Max. of u, N		
		FC_Y	FC_B	FC
(0.1, 1, 1)	$\theta(0)$ = 30°	~50	50.00	38.01
(0.1, 1, 1)	$x(0)$ = 2m	~3	25.00	11.03
(0.1, 0.2, 1)	$\theta(0)$ = 30°	~50	50.00	38.05
(0.5, 1, 1)	$\theta(0)$ = 30°	~50	50.00	38.00
Mean value		38.25	43.75	31.28
Variation, %		0.00	14.38	-18.23

It can be seen from Fig. 9 that the pendulum's maximum initial deflection angles when using FC are much larger than those when using FC_Y and FC_B. The increase of this parameter when using FC compared to FC_Y and FC_B is up to 50% and 37.5% for the case of a pendulum length of 0.7m. The pendulum's maximum initial deflection angle when using FC is up to 77°. Hence, it can be seen that FC gives superior results compared to FC_Y and FC_B in the comparative cases regarding system equilibration time, maximum control force, and maximum initial deflection angle versus the pendulum length.

Third, the system's equilibrium time t_s (s) using FC with the change of the pendulum's length is shown in Fig. 10. Here the initial conditions in the simulations in Fig. 10 are the maximum initial deflection angle, as shown in Fig. 9. It is noted that the controllers FC_Y and FC_B do not have similar results for the simulations shown in Fig. 10 for comparison. It can be seen from these investigation cases that the equilibration time is less than 10s. The case l_p = 1.5 m has the smallest equilibrium time t_s = 7.25 s. On the contrary, the case l_p = 1.9 m has the largest equilibrium time t_s = 9.66 s.

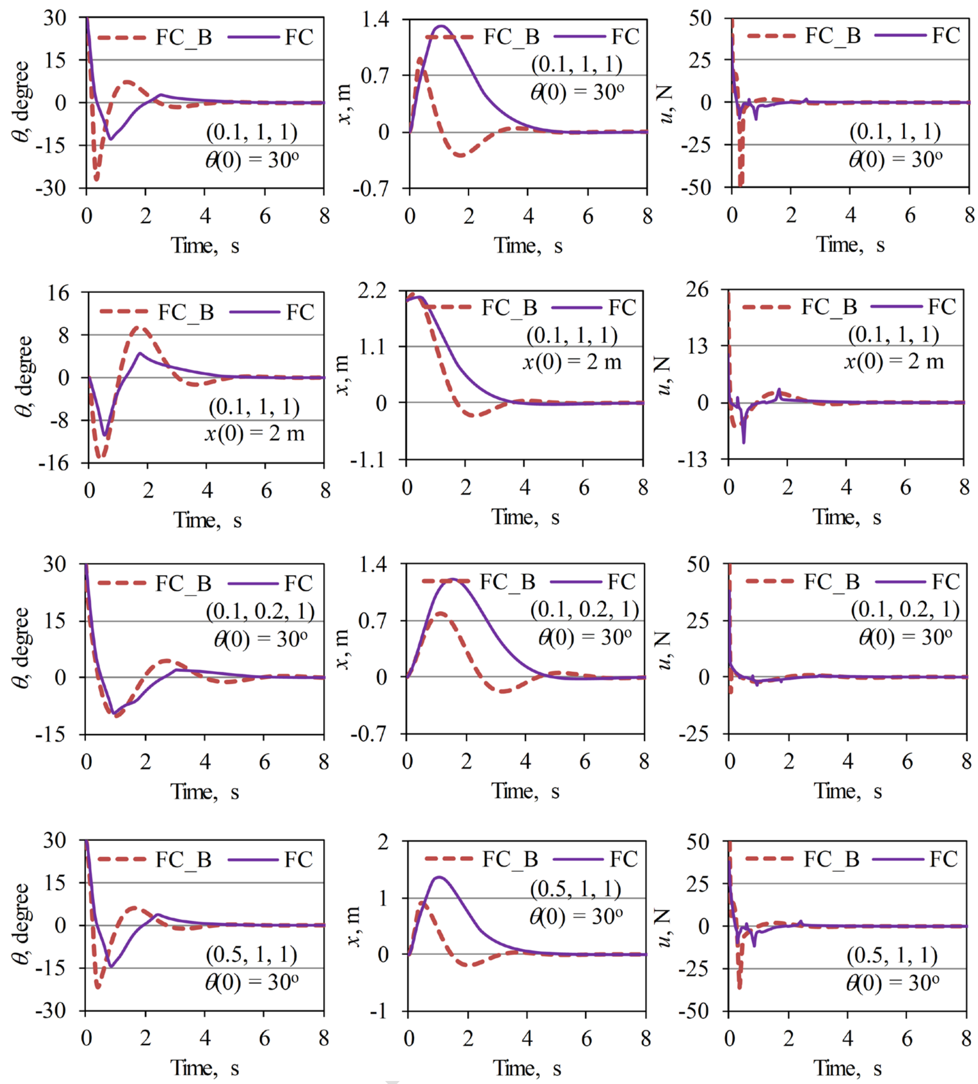

Fig. 8. Time responses of simulation cases in Table 5

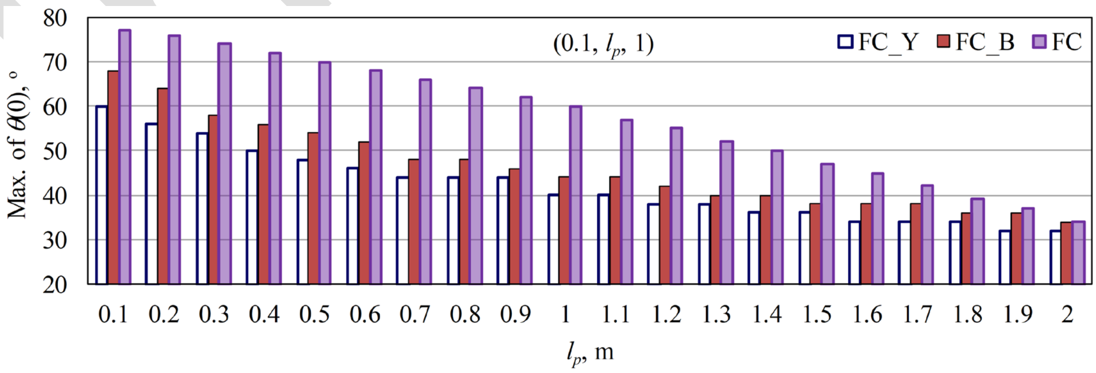

Fig. 9. Max. of $\theta(0)$ vs. pendulum's length

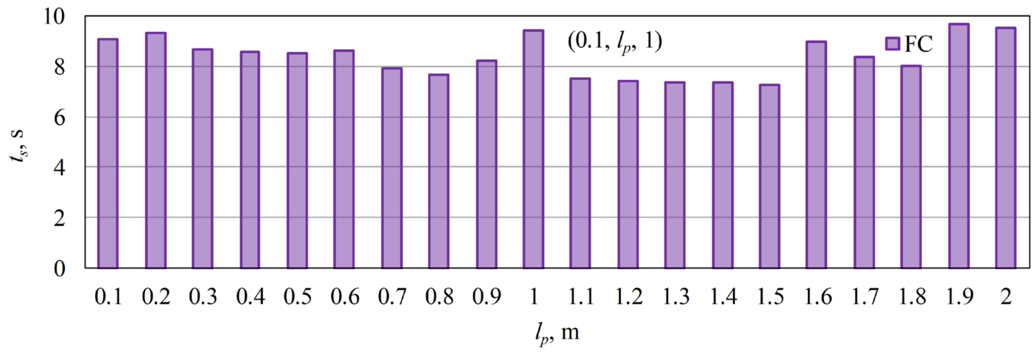

Fig. 10. Stability time (t_s, s) vs. pendulum's length

The time response of the case (m_p, l_p, m_c) equal (0.1, 0.1, 1) with the initial condition $\theta(0)$ equal 77° is shown in Fig. 11. In this case, the equilibrium time t_s is about 9.06s. With the basic configuration (m_p, l_p, m_c) equal (0.1, 1, 1), the pendulum's initial deflection angle $\theta(0)$ can be up to 60° with the equilibrium time t_s of approximately 9.41s. In this case, the system's time response is shown in Fig. 12.

The simulation results in this part show the proposed controller's stability, robustness, and high control efficiency.

Next, the determination of the maximum value of the pendulum's mass and length and the cart's mass with the pendulum's initial deflection angle $\theta(0)$ equal 30° is also carried out and shown in figures 13-15.

The pendulum's mass can be up to 2.1 kg when the pendulum's length is 1 m, and the cart's mass is 1 kg. In this case, the stable time t_s is about 10.97s, as plotted in Fig. 13. When the mass of the pendulum and the cart is 0.1 kg and 1 kg, respectively, the pendulum's maximum length can be up to 2.1 m with the equilibrium time t_s about 9.39 s, see Fig. 14. Similarly, Fig. 15 shows the case that the mass and length of the pendulum are 0.1 kg and 1 m, respectively, the cart's mass can be up to 2.6 kg with the balance time t_s about 13.12s. These results also confirm the advantages of FC, as commented in the above simulations.

It can be seen from the design steps of FC in Section 3 and simulation results in Section 4 that the proposed controller in this study is simple in setup and optimization. The proposed controller optimized by the BCMO algorithm has a faster stabilization time (t_s) and a lower maximum control force, on average, compared to FC_B and FC_Y with different initial conditions (see Table 5). In addition, the initial maximum deflection angle of the pendulum following its length in the FC case is also significantly higher than that in the FC_B and FC_Y cases. These results show that FC (with parameters optimized by the BCMO algorithm) is more efficient, stable, and robust than FC_B (with parameters determined by the trial-and-error method).

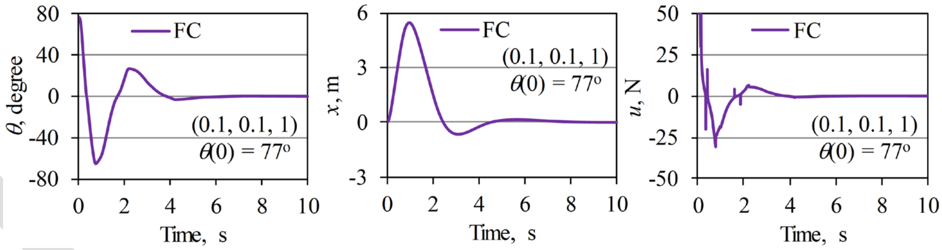

Fig. 11. Time response of the case (m_p, l_p, m_c) = (0.1, 0.1, 1), $\theta(0)$ = 77°.

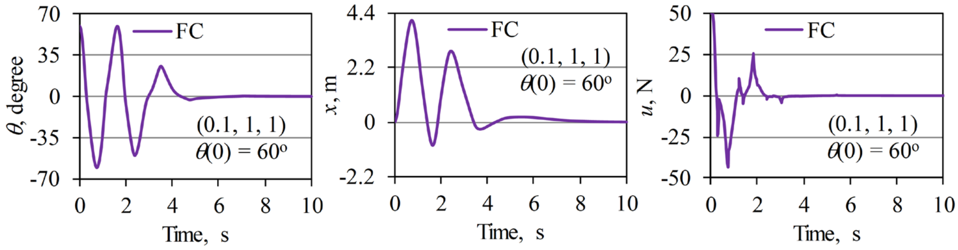

Fig. 12. Time response of the case (m_p, l_p, m_c) = (0.1, 1, 1), $\theta(0)$ = 60°

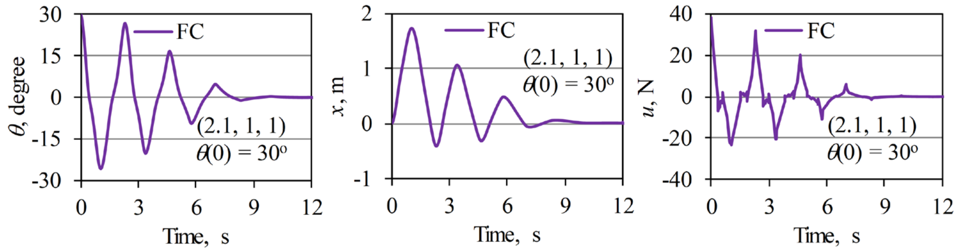

Fig. 13. Time response of the case (m_p, l_p, m_c) = (2.1, 1, 1), $\theta(0) = 30^o$

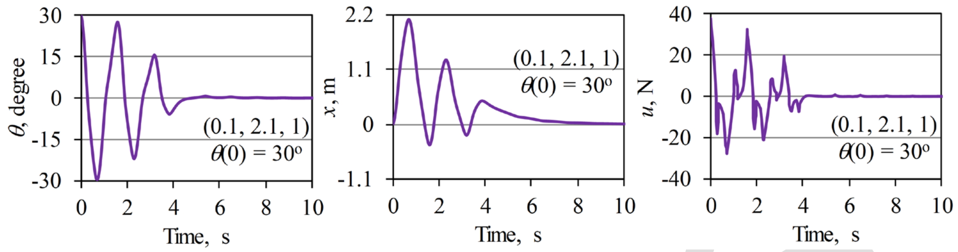

Fig. 14. Time response of the case (m_p, l_p, m_c) = (0.1, 2.1, 1), $\theta(0) = 30^o$

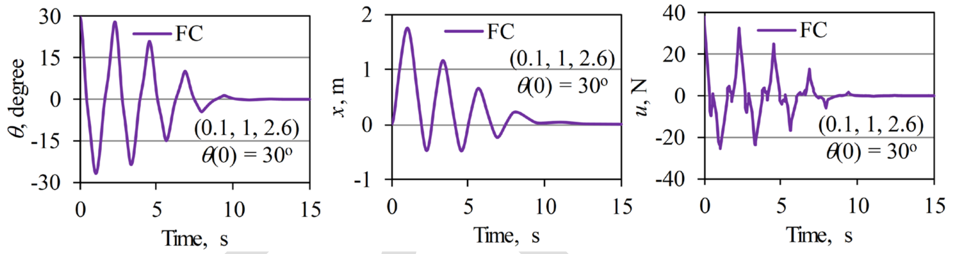

Fig. 15. Time response of the case (m_p, l_p, m_c) = (0.1, 1, 2.6), $\theta(0) = 30^o$

Underactuated systems are pretty common in mechanics and robotics when the number of the system's degrees of freedom to be controlled is greater than the number of actuators [27]. Hence, the approach in the present work is useful and can be applied to control many different underactuated mechanical models. For models of the Furuta pendulum or rotary inverted pendulum, the inertia wheel pendulum, the translational oscillator with a rotational actuator, the Acrobot, the Pendubot, the ball and beam, and the convey-crane, this approach can be applied directly to each component controller for each degree of freedom of each model. The proposed method may have to be modified for robot models or autonomous guided vehicles because the number of component controllers can increase, and the number of inputs and outputs of each component controller can change.

5. Conclusion

This study proposed a simple approach for optimizing the fuzzy controller for the underactuated inverted pendulum-cart system. The control rule bases of component controllers can be easily established based on independent observation of the influence of the control force on the state of the pendulum and that of the cart. Thus, this approach is suitable for designing fuzzy controllers of underactuated mechanical systems and robot models. The method of combining component control forces to obtain the overall control force in the paper demonstrated the proposed controller's adaptability. Finally, the simulation results show that the fuzzy controller had high efficiency and was stable and robust for the initial conditions as well as for different configurations of the system. However, the limitation of this study was that determining the combination weight w of sub-fuzzy controllers was simple and contained few parameters to improve the effectiveness of the combination.

Applying this proposed approach to control other underactuated mechanical systems, robot models, and autonomous guided vehicles is necessary for further investigations of this research direction. The combination weight needs to be improved to increase the efficiency and adaptability of the controller. Furthermore, comparison investigations with other widely used control techniques, such as PID, LQR, sliding mode, adaptive control, or neuro-fuzzy methods, and experimental studies for the investigated model are essential to verify the proposed controller's advantages thoroughly.

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References

[1] X.-T. Nguyen, T.-L. Bui, H.-A. Bui, D.-A. Pham, N. Miura, Control the Movement of Mobile Robot Using Fingers Gestures Based on Fuzzy Logic, in: Proceedings of the International Conference on Advanced Mechanical Engineering, Automation, and Sustainable Development 2021 (AMAS2021), Springer, 2022, pp. 799-804. https://doi.org/10.1007/978-3-030-99666-6_116

[2] C.A. Ibanez, O.G. Frias, M.S. Castanon, Lyapunov-based controller for the inverted pendulum cart system, Nonlinear Dynamics, vol. 40, no. 4, pp. 367-374, 2005. https://doi.org/10.1007/s11071-005-7290-y

[3] A.V. Pesterev, Y.V. Morozov, Stabilization of a cart with inverted pendulum, Automation and Remote Control, vol. 83, no. 1, pp. 78-91, 2022. https://doi.org/10.1134/S0005117922010064

[4] S.K. Mishra, D. Chandra, Stabilization and tracking control of inverted pendulum using fractional order PID controllers, Journal of Engineering, vol. 2014, no. 1, pp. 752918, 2014. https://doi.org/10.1155/2014/752918

[5] S. Irfan, A. Mehmood, M.T. Razzaq, J. Iqbal, Advanced sliding mode control techniques for inverted pendulum: Modelling and simulation, Engineering Science and Technology, an International Journal, vol. 21, no. 4, pp. 753-759, 2018. https://doi.org/10.1016/j.jestch.2018.06.010

[6] S. Hanwate, Y.V. Hote, A. Budhraja, Design and implementation of adaptive control logic for cart-inverted pendulum system, Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering, vol. 233, no. 2, pp. 164-178, 2019. https://doi.org/10.1177/0959651818788148

[7] M. Ashok Kumar, S. Kanthalakshmi, H∞ tracking control for an inverted pendulum, Journal of Vibration and Control, vol. 24, no. 16, pp. 3515-3524, 2018. https://doi.org/10.1177/1077546317750977

[8] M. Abdullah, A.A. Amin, S. Iqbal, K. Mahmood-ul-Hasan, Swing up and stabilization control of rotary inverted pendulum based on energy balance, fuzzy logic, and LQR controllers, Measurement and Control, vol. 54, no. 9-10, pp. 1356-1370, 2021. https://doi.org/10.1177/00202940211035406

[9] H.-Y. Zhang, J. Wang, G.-D. Lu, Self-organizing fuzzy optimal control for under-actuated systems, Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering, vol. 228, no. 8, pp. 578-590, 2014. https://doi.org/10.1177/0959651814533681

[10] Z. Ping, M. Zhou, C. Liu, Y. Huang, M. Yu, J.-G. Lu, An improved neural network tracking control strategy for linear motor-driven inverted pendulum on a cart and experimental study, Neural Computing and Applications, vol. 34, no. 7, pp. 5161-5168, 2022.

[11] U. Riaz, A.A. Amin, M. Tayyeb, Design of active fault-tolerant control system for Air-fuel ratio control of internal combustion engines using fuzzy logic controller, Science Progress, vol. 105, no. 2, pp., 2022. https://doi.org/10.1177/00368504221094723

[12] J. Lin, Y.S. Ding, J. Chang, Balancing and swinging-up control for cart-pendulum-seesaw system by decomposed fuzzy coordination control, Journal of Vibration and Control, vol. 20, no. 6, pp. 925-942, 2014. https://doi.org/10.1177/1077546312466885

[13] B.A. Elsayed, M.A. Hassan, S. Mekhilef, Fuzzy swinging-up with sliding mode control for third order cart-inverted pendulum system, International Journal of Control, Automation, and Systems, vol. 13, no. 1, pp. 238-248, 2015. https://doi.org/10.1007/s12555-014-0033-4

[14] A.I. Roose, S. Yahya, H. Al-Rizzo, Fuzzy-logic control of an inverted pendulum on a cart, Computers and Electrical Engineering, vol. 61, no., pp. 31-47, 2017. https://doi.org/10.1016/j.compeleceng.2017.05.016

[15] T.B. Ivanova, A.A. Kilin, E.N. Pivovarova, Controlled motion of a spherical robot with feedback. II, Journal of Dynamical and Control Systems, vol. 25, no. 1, pp. 1-16, 2019. https://doi.org/10.1007/s10883-017-9390-7

[16] B. Elkinany, M. Alfidi, R. Chaibi, Z. Chalh, TS fuzzy system controller for stabilizing the double inverted pendulum, Advances in Fuzzy Systems, vol. 2020, no., pp. 1-9, 2020. https://doi.org/10.1155/2020/8835511

[17] E. Kelekci, T. Yaren, S. Kizir, Design of optimized interval type-2 fuzzy logic controller based on the continuity, monotonicity, and smoothness properties for a cart-pole inverted pendulum system, Transactions of the Institute of Measurement and Control, vol. no., pp. 2291–2307, 2022. https://doi.org/10.1177/01423312221081309

[18] C.-H. Chiu, Y.-T. Hung, Y.-F. Peng, Design of a decoupling fuzzy control scheme for omnidirectional inverted pendulum real-world control, IEEE Access, vol. 9, no., pp. 26083-26092, 2021. https://doi.org/10.1109/ACCESS.2021.3057658

[19] A. Jain, A. Sharma, V. Jately, B. Azzopardi, S. Choudhury, Real-time swing-up control of non-linear inverted pendulum using Lyapunov based optimized fuzzy logic control, IEEE Access, vol. 9, no., pp. 50715-50726, 2021. https://doi.org/10.1109/ACCESS.2021.3058645

[20] A.N.K. Nasir, A.A.A. Razak, Opposition-based spiral dynamic algorithm with an application to optimize type-2 fuzzy control for an inverted pendulum system, Expert Systems with Applications, vol. 195, no., pp. 116661, 2022.

[21] T. Abut, S. Soyguder, Two-loop controller design and implementations for an inverted pendulum system with optimal self-adaptive fuzzy proportional integral derivative control, Transactions of the Institute of Measurement and Control, vol. 44, no. 2, pp. 468-483, 2022. https://doi.org/10.1177/01423312211040301

[22] S.M. Arbatsofla, A.H. Mazinan, M. Mahmoodabadi, An optimal fuzzy fractional-order adaptive robust controller according to feedback linearization for an under-actuated nonlinear inverted pendulum system, International Journal of Dynamics and Control, vol. 13, no. 3, pp. 107, 2025. https://doi.org/10.1007/s40435-025-01611-y

[23] J. Yi, N. Yubazaki, Stabilization fuzzy control of inverted pendulum systems, Artificial Intelligence in Engineering, vol. 14, no. 2, pp. 153-163, 2000. https://doi.org/10.1016/S0954-1810(00)00007-8

[24] J.C. Butcher, Numerical methods for ordinary differential equations, John Wiley & Sons, 2016. https://doi.org/10.1002/9781119121534

[25] H.-L. Bui, M.-N. Pham, T.-T.-H. Nguyen, Swing-up control of an inverted pendulum cart system using the approach of Hedge-algebras theory, Soft Computing, vol. 26, no. 10, pp. 4613-4627, 2022. https://doi.org/10.1007/s00500-022-06968-2

[26] T. Le-Duc, Q.-H. Nguyen, H. Nguyen-Xuan, Balancing composite motion optimization, Information Sciences, vol. 520, no., pp. 250-270, 2020. https://doi.org/10.1016/j.ins.2020.02.013

[27] J. Moreno-Valenzuela, C. Aguilar-Avelar, Motion control of underactuated mechanical systems, Springer, 2018. https://doi.org/10.1007/978-3-319-58319-8