

# Auto-Tuning Parameters of the Offline Optimal Motion Cueing Algorithm with Mean-Variance Mapping Optimization

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## Abstract

A motion cueing algorithm (MCA) not only maintains the simulator within its physical limits but also generates such movements of the driving simulator that the necessary motion cues of drivers on the realistic vehicle are equivalently reproduced. The offline optimal MCA focuses on finding the best combination of the translational acceleration and tilt angles of the motion platform to maintain drivers' motion perception. However, the best combination depends on the MCA's parameters, tuned mainly by trial and error with experts in the loop. Moreover, for different amplitude input signals, the parameters are accordingly modified. This manual tuning procedure is so time-consuming that the generic optimization, named Mean-Variance mapping optimization, was proposed to search the suitable parameters for the optimal algorithm. This tuning method uses the specific cost function of constraint conditions such as workspace limits, avoiding false cues, and improving motion fidelity to achieve the best parameters for optimal MCA with the particular input signals.

Keywords: Motion cueing algorithm, Mean-Variance mapping optimization method, false cues.

## 1. Introduction

The motion cueing algorithms aim to preserve the perceptual realism of the simulation by using tilt coordination to mimic sustained translational acceleration. The classical MCA is first developed by Conrad and Schmidt [1], then several MCAs have been developed with a different optimal technique [1-4]. The tuning of the MCA is complicated due to the intransparent parameters of the MCA. Most tuning techniques were introduced for the classical algorithm to solve the intransparent problem [5, 6]. Moreover, the numerous parameters of the MCAs and fitting characteristics of human motion perception are also complex tuning tasks. Hence, the overall goal of the tuning is to improve the perceived motion inside the simulator to resemble the feeling of driving a real car and avoid simulator sickness at the same time.

Further, a driver or pilot-in-the-loop tuning as suggested by Grant and Reid [7] is very time-consuming. Thus it has to be investigated whether other tuning methods, such as maneuver-specific tuning, can be satisfactory as well. However, the parameters of the MCAs have severe effects on the simulated motion that can lead to motion sickness if the motion and visual cues are not consistent. Thus, for particular motion simulation, tuning the best parameters of an MCA remains a challenging task [7]. Moreover, an open question remains whether well-tuned classical algorithms are sufficient or whether complex new strategies have to be developed

In the literature, the first tuning is to find the appropriate parameters with which the MCAs generate the simulated quantities in the defined ranges. For a specific drive task, the tuning processes were fulfilled mostly by trial and error; thus, it is very time-consuming and requires drivers' experience in the simulation field [8, 9]. The methods used fuzzy control theory to constrain angular velocity false cues and the position of the driving simulator in the motion platform's workspace.

After finding the suitable parameters, the objective and subjective assessments were implemented for extensive tuning the MCAs to achieve realistic simulation. The tuning process is also done by trial and error with experts in-the-loop, and the optimal parameters depend on the specification of driving simulators and simulation tasks, etc. For example, an MCA is objectively tuned for generating necessary motion cues and eliminating the false cues. In another approach, the participant can subjectively evaluate the level of realism of a driving simulation based on the scores, or the statements were given after taking the simulation task, then the parameters will be modified to bring a better experience to the drivers. Because of the lack of information of the human receptors, the tuning process including two stages is necessary. For example, Reid and Nahon [3] firstly used the objective assessment for roughly tuning, then the subjective assessment for fine-tuning for the MCAs in the flight simulator.

Besides, the fidelity criteria for objective assessment of the simulated quantities were inferred from the subjective assessment. The criteria can save time-consuming for trial and error and be investigated in some research related to flight simulation [9, 10]. Therefore, the tuning of the parameters is an open issue because the parameters are not intuitive for inexperienced simulator users. The optimal tracking or the MPC strategy were applied to correspond explicitly to the motion system physical limits and motion detection thresholds. However, automated tuning would be desirable in order to ease the user to implement the time-consuming task.

This work's main objectives are 1) definition of the well-tuned index tuning procedure, 2) designation of automatic tuning procedures for any motion task and kinds of the motion platform, 3) application of the Auto-Tuning procedures for an offline optimal algorithm, and 4), implementation of the strategies for the offline optimal MCA for the various amplitude of input signals.

In the next section, the offline optimal MCA is firstly introduced. Secondly, the Auto-Tuning method (AT) and its cost functions are explained in detail. Thirdly, an example that applied the AT method for offline optimal MCA with various amplitude of input signals is implemented. Lastly, the results are discussed in the conclusion section.

## 2. Numerical Method Assessing the Well-Tuned Parameters for an MCA

Table 1. Priority levels for tuning process

Order	Statement in the literature
1st	<ul style="list-style-type: none"> <li>- False cues give large decrease of the motion fidelity [7] ;</li> <li>- Distortions in the reproduction of motion cues may even have more adverse effect than not having motion cues [13,14];</li> <li>- Opposite motion cues, and false lateral specific force due to the cockpit rotation make motion fidelity even lower than in the case of fixed-base simulation [15].</li> </ul>
2nd	<ul style="list-style-type: none"> <li>Phase lag due to the low-pass filter gives the significant reduce of motion fidelity [7];</li> <li>-Missing cues is the serious case of scale errors [7].</li> </ul>
3rd	Down-scaling the specific force is suggested to be the most desirable [16].
4th	The well-tuned parameter for classical algorithm could give the similar perception as lane position algorithm, which uses the tilt-coordination when there is no better choice [17]

In the literature (Table. 1), an MCA parameters are tuned to generate the suitable motions inside the restricted workspace of a driving simulator and have no false cues as well as provide as much as benefit motion cues.

Based on the statements in the literature, the significant effect of the errors of motion cues and usage of the workspace are clear seen. In order to satisfy all the tuning criteria for a MCA, in some simulation case, is impossible. Therefore, the priority order, that classifies the significance into 4 levels, is given such as:

(1st) The false cues → (2nd) missing cues, phase lag → (3rd) scale errors → (4th) using more translational movement The order is based on the statements in the literature, that emphasized the significant effect of the false cues, missing cues on the motion fidelity. Moreover, the quite clear effect of the scale factor and using more translational motion for reproducing linear acceleration are considered.

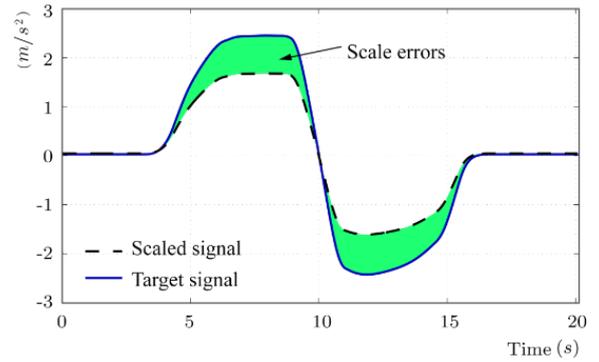


Fig. 1. Scale error of the simulated acceleration

The scale error (Fig. 1) describing the strong motion reproduced in the driving simulator is defined as  $e_{sc} = \frac{\int a_{sy} dt}{\int a_{vy} dt}$ , where  $a_{sy}$ , dash black line represents simulated acceleration and  $a_{vy}$ , solid blue. line represents target acceleration.

In this works, a proposed method classifies the level of well-tuned parameters according to the criteria prioritized  $G_i$  with  $i = \{1..4\}$  (1st: being implementable inside the workspace; 2nd: having no false cues, 3rd: generating suitable scale simulated signal (related to the small scale error,  $e_{sc}$  Fig. 1); and 4th: generating more translation motion the rotational motion). The well-tuned index is computed as

$$WI = 2^3 G_1 + 2^2 G_2 + 2^1 G_3 + 2^0 G_4. \quad (1)$$

where

$G_1 = 1$ : means no violation and a proper condition for tilt-coordination

$G_2 = 1$ : means no false cues due to either specific force distortion or extra rotational cues

$G_3 = 1$ : means to avoid too weak motion cues

$G_4 = 1$ : means to using as much translational movement as possible to simulate lateral acceleration

The priority is described as levels of a binary number in which the high level is obtained if all criteria are satisfied. The worst level is for the occurring false cues, and the violation situation occurs if the first criterion is not satisfied. The criteria  $G_i$  can be applied to distinguish three levels: High (WI = 14-15), Medium (WI = 12-13), Low (WI=7-10). If WI < 8 there is limit violation.

The formulations for checking the criteria are showed in the Table 2.

Table 2. The criteria formulation

Criteria	Formulations
$G_1 = 1$	$S_{Sy} \leq S_{max}; \alpha_S \leq 30^0;$
$G_2 = 1$	$\omega_{Sx} \leq \omega_{th}; \dot{\omega}_{Sx} \leq \dot{\omega}_{th}; h \cdot \dot{\omega}_{th} \leq k_{fT};$ $\frac{e_{f,sh}^*}{\delta_0} \leq k_e$
$G_3 = 1$	$k_S \in [k_{S,min}, k_{S,max}]$
$G_4 = 1$	$\frac{\int a_{Sy} dt}{\int a_{Vy} dt} \geq k_{a,min}$

where:

$S_{Sy}$  and  $S_{max}$  are the simulated position and the limitation of the workspace, respectively

$\alpha_S$  is the tilt angles due to the tilt coordination technique.

$\omega_{Sx}$  and  $\dot{\omega}_{Sx}$  are the simulated angular velocity and acceleration

$\omega_{th}$  and  $\dot{\omega}_{th}$  are the thresholds values of angular velocity and acceleration.

$e_{f,sh}^*$  is the maximum of shape errors (Fig. 2) of simulated specific force regarding to target one.

$k_{f,T}$  is the maximum value of specific force ( $f = a - g$ ,  $a$  is the acceleration and  $g$  is the gravity acceleration) causing by tilt angular acceleration

$k_e$  is the maximum ratio of shape error to the threshold of otolith system

$h$  is the distance between the center of rotation with the drivers head.

$k_{a,min}$  is the minimum ratio of simulated translational acceleration to the target one.

$k_S$  is the global scale factor of simulated specific force.  $a_{Sy}, a_{Vy}$ : are the simulated and target acceleration, respectively.

The shape error is defined as  $e_{sh} = ka_{vy} - a_{sy}$ , with  $k$  scaled down factor regarding target signal. The larger factor could lead to more false cues during simulation.

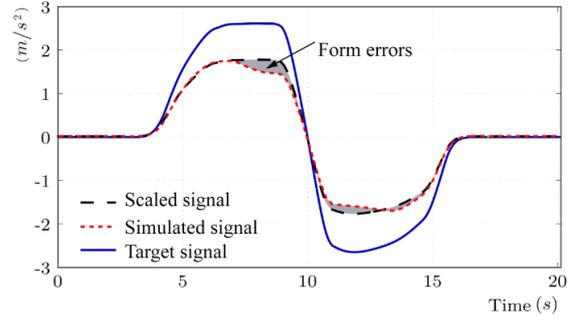


Fig. 2. Shape error of the simulated acceleration

Table 3. Boundary values of the criteria [10]

$\delta_0$	$\omega_{th}$	$\dot{\omega}_{th}$	$k_{S,min}$
0.17(deg/s²)	6(deg/s)	11(deg/s²)	0.36
$k_{S,max}$	$k_{a,min}$	$k_{fT}$	$k_e$
1	0.50	0.1(m/s²)	1

The boundary values for the criteria can be reviewed from the related experiments in the literature or from the specific investigation for a specific maneuver task. In the paper, all the boundary values showed in Table. 3 are gathered from the publications that are in detail listed in the Pham (2017) [10]. Note that, the criteria are particularly developed for the Rollercoaster simulation running along planar S-curve with only lateral acceleration, but the criteria could be applied for other situations with the change of suitable parameters.

### 3. Offline Optimal Motion Cueing Algorithm

The algorithm (Fig. 3), first introduced by Zywiol and Romano [4], solves the problem as an optimal tracking problem, whose cost function in (1) that aims to find the appropriate time history for the control input vector  $\mathbf{u} = [u_1, u_2]^T$  to make the output signal  $\mathbf{y} = [y_1, y_2]^T$  track the reference output vector, which is  $\mathbf{y}_r = [f_r, 0]^T$ .

The output motion includes translational acceleration  $a_S$  and tilt angle  $\varphi_S$  that are used to compute the simulated translational acceleration  $a_S$  and tilt angle  $\varphi_S$  that are used to compute the simulated specific force  $f_S$  perceived by human's vestibular system.

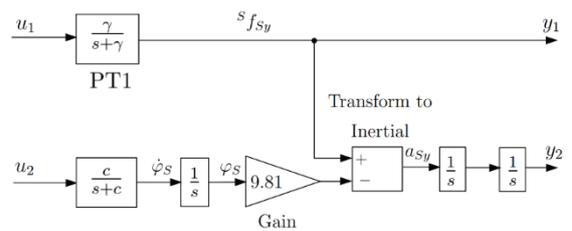


Fig. 3. Diagram of offline optimal MCA

$$\mathbf{J} = \int_{t_0}^{t_f} [(\mathbf{y} - \mathbf{y}_r)^T \mathbf{Q}(\mathbf{y} - \mathbf{y}_r) + \mathbf{u}^T \mathbf{R} \mathbf{u}] dt. \quad (2)$$

where  $\mathbf{Q}_r = \text{diag}\{q_1, q_2\}$  and  $\mathbf{R} = \text{diag}\{r_1, r_2\}$  are the weighting matrices. Moreover, the two first-order filters PT1 are added in both channels (Fig. 3) to smoothen the simulated signals. Therefore, the algorithm has six parameters  $\{q_1, q_2, r_1, r_2, c, \gamma\}$  that must be tuned appropriately for generating suitable motion.

#### 4. The Auto-Tuning Method with Mean Variant Mapping Optimization

Mean-Variance mapping optimization (MVMO), a population-based stochastic optimization technique developed by Erlich *et al.*, mixes the good performance of a specific number of best individuals to achieve an expected better generation for the following optimizing stages [12]. This method uses its unique transformation strategy for mutated genes of the offsprings based on the Mean-Variance of the n-best population by employing the concepts of selection, mutation, and crossover from evolutionary computation algorithms.

Table 4. Functions of cost functions for the auto-tuning procedure

Penalty function	Formulation
Position	$J_P = \begin{cases} 0 & \text{No violation} \\ \sum (e^{(\text{Min}(P)-L)^2} - 1) + (e^{(\text{Max}(P)-U)^2} - 1) & \text{Violation} \end{cases}$ <p><math>\text{Min}(P), \text{Max}(P)</math> represents the minimum / maximum position of the motion platform, and <math>L / U</math> represents the lower / upper physical limit of the motion platform.</p>
Shape error	$J_{esh} = \begin{cases} 0 & \text{if } \max  e_{fsh}  < \delta_0 \\ e^{(\max  e_{fsh}  - \delta_0)^2} - 1 & \text{if } \max  e_{fsh}  \geq \delta_0 \end{cases}$ <p><math>\delta_0</math> is the otolith threshold value.</p>
Angular velocity	$J_\omega = \begin{cases} 0 & \text{if }  \omega_{max}  < \delta_s \\ e^{(\omega_{max} - \delta_s)^2} - 1 & \text{if }  \omega_{max}  \geq \delta_s \end{cases}$ <p><math> \omega_{max} </math> is the maximum of the absolute simulated angular velocity, and <math>\delta_s</math> is threshold value of the angular velocity perceived by the semicircular organ.</p>
Angular acceleration	$J_{\dot{\omega}} = J_\omega = \begin{cases} 0 & \text{if }  \dot{\omega}_{max}  < \dot{\omega}_s \\ e^{(\dot{\omega}_{max} - \dot{\omega}_s)^2} - 1 & \text{if }  \dot{\omega}_{max}  \geq \dot{\omega}_s \end{cases}$ <p><math>\dot{\omega}_{max}</math> is the maximum of the absolute simulated angular acceleration, and <math>\dot{\omega}_s</math> is threshold value of the angular acceleration perceived by the semicircular organ.</p>
Scale error	$J_{sc} = \begin{cases} k_{jsc} (e^{(k_s - k_s^{max})^2} - 1) & \text{if } k_s \in [k_s^{min}, k_s^{max}] \\ e^{(k_s - k_s^{min})^2} - 1 & \text{if } k_s < k_s^{min} \\ e^{(k_s - k_s^{min})^2} - 1 & \text{if } k_s > k_s^{min} \end{cases}$ <p><math>[k_s^{min}, k_s^{max}]</math> represents the range of the desired scale factors. The factor <math>k_{jsc}</math> reduces the values of <math>J_{sc}</math> when is in the allowed range compared to those when <math>k_s</math> is outside.</p>
Translational motion	$J_{tr} = \begin{cases} 0 & \text{if } k_a \leq k_a^{min} \\ e^{(k_a - k_a^{min})^2} & \text{if } k_a > k_a^{min} \end{cases}$ <p><math>k_a^{min}</math> denotes the minimum desired ratio of translational movement to reproduce the translational specific force.</p>
Washout	$J_{wo} = \begin{cases} 0 & \text{if } \varphi_s(t_f) = 0 \\ e^{(g\varphi_s(t_f))^2} - 1 & \text{if } \varphi_s(t_f) \neq 0 \end{cases}$ <p><math>\varphi_s(t_f)</math> denotes the final tilt angle and <math>g</math> is the gravity acceleration.</p>
Note	<p><math>k_s</math> : The ratio of simulated signal to target signal. <math>k_a</math> : The ratio of translational motion to target motion</p>

The parameters of the offline optimal algorithm are tuned to achieve the high well-tuned index MF. Thus, the cost function of the Auto-Tuning process with the MVMO method represents the combination of the penalty functions  $J_O$  (see Table 4. for a full explanation) defined as:

$$F_{MVMO} = \sum_{i \in \{P,esh,\omega,\omega,sc,tr,Wo\}} W_k J_k. \quad (3)$$

The selected cost function  $J_O$  is to solve all the tuning problems (Fig. 4) related to 1) limited workspace; 2) constraining scale factor; 3) type of false cues; 4) defined good levels; 5) amount of motion errors. The cost function, which combines the advantage of the numerical index [11] (Fig. 4a) and the well-tuned index [12] (Fig. 4b), is aimed to pull the simulated signals under threshold values of motion perception, such as acceleration and angular velocity. As presented in Fig. 4a, the numerical index criterion  $\lambda_{m,n}$ , where  $m = \{1f, 1\omega\}$  and  $n = \{sh, sc\}$  represent the numerical indices related to specific force error ( $f$ ) and angular velocity errors ( $\omega$ ). Therefore, it is hard to find suitable parameters even if the numerical indices have small total values and components. On the other hand, Fig. 4b presents well-tuned index proposed by Duc-An with 4 levels of MF including good, medium, low, existing violation case [10]. The well-tuned indices provide the sign of appearing the false cues type, limiting condition and defined good levels by using the working and good perception boundary.

The penalty function, for example the angular velocity penalty function demonstrated in Fig. 4c, considers only the amount of motion error of the

simulated quantities regarding to target one, other problems: limit workspace, type of false cues, constraining scale factor. The penalty functions use the exponential function to pull the maximum angular velocity under its threshold values. The technique are applied to other quantities such as: scale factor, limit boundary, and for washout technique. Therefore, the Auto-Tuning process with the penalty function can find the suitable parameters that reduce all false cues, and avoid violation with physical limits and exploit the linear motion of the motion platform

### 5. Applying Auto-Tuning Method for the Optimal Tracking Algorithm

This section describes the application of the Auto-Tuning methods to find the suitable parameters for ZyRo algorithm to generate the maximum global scale factor [10]. A test case for Auto-Tuning parameters is a lateral acceleration,  $a$ , of a ride with constant velocity  $v = 3.6$  (m/s) of a virtual roller coaster along the planar S-curve rail (Fig. 5)

The result of the test case in reference [10] proved the feasibility of the Auto-Tuning method. Concretely, as listed in Table. 5, the maximum global scale factor is  $k_s^* = 0.92$ . Considering the false cues, all angular velocities and accelerations ( $\omega_s$  and  $\dot{\omega}_s$ ) are under their threshold values (Fig. 5) due to the penalty function  $J_\omega$  and  $J_{\dot{\omega}}$ . Note that the penalty function  $J_\omega$  constrains the angular velocity to reach its threshold. This property is hard to obtain if the parameters of the ZyRo algorithm are manually tuned.

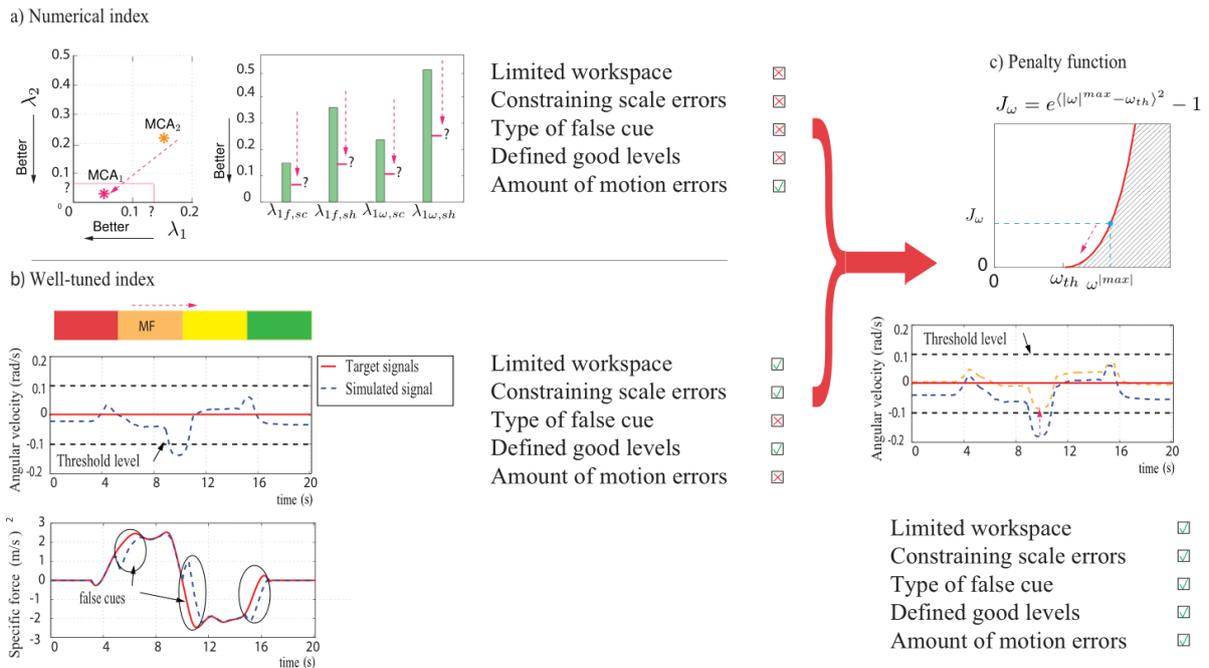


Fig. 4. Principle of Auto-Tuning method a) numerical indices, b) Well-tuned indices, c) Penalty functions

Moreover, the simulated specific forces  $f_{Sy}$  tracks well the target  $f_{Vy}$  with small shape errors  $e_{sh} = 0.018$  that is much smaller than the otolith threshold  $\delta_O$ . Fig. 6 shows two elements of the simulated specific force which are tilted acceleration  $a_{Ty}$  and linear acceleration  $a_{Sy}$ . Thanks to the washout penalty function  $J_{W_o}$ , the simulated angle  $\varphi_S$  which is pulled to zero at the end of the simulation cause the  $a_{Ty}$  come to zero at the moment. By using Auto-Tuning parameters, the algorithm generates more tilt angles to compensate for the large acceleration.

Table 5. Optimal values of the parameters ([10])

$q_1$	$q_2$	$r_1$	$r_2$
[0.001,0.1]	[1,100]	[1,300]	[0.001,10]
0.0553	93.1597	57.0096	7.4812
$c$	$\gamma$	$k_S^*$	$WI$
[0.001,1]	[0.01, 1]	[0.4,1]	-
0.8169	0.0229	0.92	14

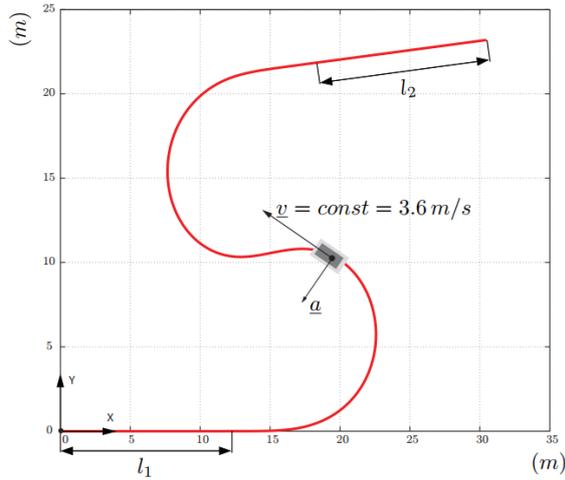


Fig. 5. S-curve trajectory of the test case with constant velocity 3.6 m/s (based on [10])

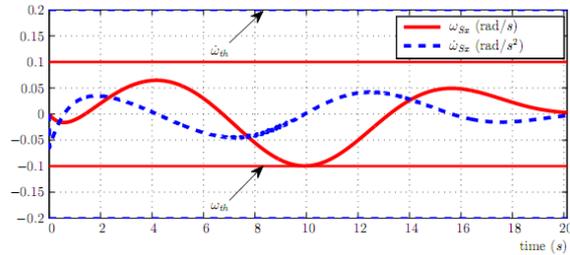


Fig. 6. Simulated angular velocity and angular acceleration (based on [10])

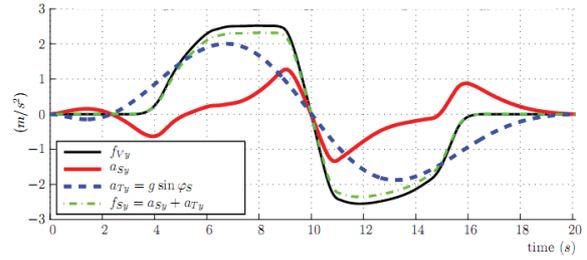


Fig. 7. Simulated specific forces and accelerations (based on [10])

The tilt angle is even generated early to prepare workspace for large linear acceleration  $a_{Sy}$ .

### 6. Auto-Tuning Parameters of Offline Optimal MCA for the Various Amplitude of Input Signals

Based on the good result of the test case in reference [10], a test case for Auto-Tuning parameters for a lateral acceleration of a ride with constant velocity  $v = 3.6$  (m/s) of a virtual roller coaster along the planar S-curve rail, the Auto-Tuning procedures are used to the tuned parameter of the optimal MCAs with various input signals that have successively increased amplitudes of  $\{v, 1.3v, 1.5v, 2.0v\}$  the amplitude of the lateral acceleration.

Moreover, each ride's threshold values were selected according to the general psychophysical Weber's law of Just Noticeable Differences: the difference is a constant proportion of the original stimulus value  $\Delta I/I = \text{constant}$ , with  $I$  represents the initial stimuli intensity,  $\Delta I$  represents the differential threshold<sup>19</sup>. The selected thresholds for different amplitudes of input signals  $\{v, 1.3v, 1.5v, 2.0v\}$  are  $\{0.1, 0.17, 0.225, 0.4\}$  (rad/s) respectively. The tuned weighting values are shown in Table 6, and the responses of the optimal MCA are shown in Fig. 5. It could see that the Auto-Tuning procedures can find the best weighting values for small velocity case. The simulated specific force closely tracks the target specific force, while the angular velocity is under threshold values. If the amplitude of velocity gradually increases with 130% - 200%, the higher amplitude of target specific force is achieved. The optimal MCA still has high well-tuned index  $MF=14$  with the case of  $1.3v$ ; however, the well-tuned index for the case of  $1.5v$  is reduced to medium level, especially for case  $2.0v$ , the  $MF$  is to low level because the simulated angular velocity is over its threshold level. Note that, at the end of the simulating period, the specific force and angular velocity are pulled to zero due to the effect of the washout penalty function  $J_{W_o}$ . The washout effect is not considered in the algorithm, but it can be fulfilled with the cost function of the auto-tuning process.

Table 6. Tuned weighting values for optimal MCA for each amplitude of input signals.

Velocity	c	$\gamma$	q <sub>1</sub>	q <sub>2</sub>	r <sub>1</sub>	r <sub>2</sub>
v	0.5821	0.1879	6.7386	100	198.5741	1
1.3 v	0.9078	0.0555	5.0235	69.4231	244.6879	104.3443
1.5 v	0.8422	0.0838	2.6850	87.2158	57.0902	164.5792
2.0 v	0.5460	0.0277	9.3471	73.5014	37.3821	220.8595

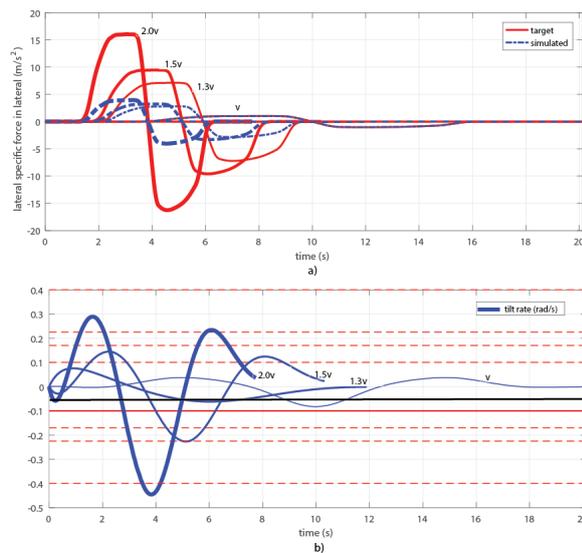


Fig. 8. Responses of optimal MCAs for different amplitude of input signals with auto-tuned parameters a) lateral specific forces, b) tilt rate – roll angular velocity

## 7. Conclusion

In summary, tuning parameters for optimal MCA for the different amplitude of input signals focus on reducing false cues and reproducing scaled specific force. This paper introduces a novel Auto-Tuning method that combines numerical index and well-tuned index for systematically tuning weighting values of the optimal MCA. Applying the Auto-Tuning procedure has several advantages compared to manual tuning methods. Firstly, the approach is flexible and saves time-consuming for tuning suitable parameters for optimal MCA to remove the false cues, especially for the large amplitude of input signals. A designer can easily adjust the weighting parameters for tuning purposes and transparently evaluate the maximum amplitude of input signals for a particular driving simulator. Secondly, the tuning method considers two main quantities: the position, angular velocity, and various physical quantities related to the human perception. Finally, in the future, the Auto-Tuning process can be applied to find the parameters for online MCAs with different drive-tracks in offline mode.

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