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Frequency Domain Based Conditions for Determining Convergence Learning Parameters in Linear Iterative Learning Control

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Abstract

This article proposes four sufficient conditions to determine the convergence learning parameters in iterative learning control of linear batch processes. These conditions are established in frequency domain with transfer functions of elementary linear learning functions of P-, D-, PD- and PID-Type, instead of their state space models as usual. Hence, they can overcome all conservative difficulties occurred by using conditions created in time domain. To obtain these conditions in frequency domain, first an overall sufficient condition belonging to input-output mapping is created, and then realized it particularly in frequency domain for four different linear learning functions by using their transfer function. The obtained conditions in frequency domain are expressed in algebraic inequality of matrix norm, so they are very convenient in use. To illustrate the applicable ability of proposed conditions in various practical applications some numerical simulations had been carried out in the paper. Obtained simulation results authenticate the advantage of these conditions.

Keywords: ILC, learning function, update law, intelligent control.

1. Introduction

Main task in control engineering is to force the output response of a controlled process to follow asymptotically a desired reference, or at least as close as possible to it. To work out this task, especially for repetitively working processes (batch processes), the iterative learning control (ILC) seems as an effective control technique, not only that ILC provides excellently an output tracking performance as required [1] but also it accomplished the close-loop control system being highly robust with unwanted disturbances [2-4]. Furthermore, with ILC the controller design does not need to have any mathematical model of processes as by applying conventional control methods. Hence, ILC belongs to intelligent control concept [3,5,6].

The [1] proposed idea of ILC to control batch processes, which operates repeatedly in a fixed time period T such as industrial robots or batch reactions in chemical processes, is that the tracking errors in outputs during each working period will be recorded for computing an appropriate modification to the input signal that will be applied to the process during next working period. In ILC concept all these operations belonging working period are often called a trial and these input refinements are carried out continuously, from trials to trials until the desired output tracking performance is reached.

If we denote each wording period (or trial) of process with the index $k = 0,1, \dots$ and the time instant among this trial with $0 \le \tau \le T$, then during the control an arbitrary controlled time $0 \le t < \infty$ will be expressed as $t = kT + \tau$. Accordingly, both vectors of inputs, outputs $\underline{u}(t)$, $\underline{y}(t)$ of a MIMO process are described as below, respectively

$$\underline{u}(t) = \underline{u}_k(\tau) \text{ and } \underline{y}(t) = \underline{y}_k(\tau). \quad (1)$$

With these notations the input refinements for next trial $k+1$ are illustrated mathematically by

$$\underline{u}_{k+1}(\tau) = \underline{u}_k(\tau) + \underline{f}_l\left(\underline{e}_k(\varsigma), \mathcal{K}\right) \quad (2)$$

with a set of appropriate parameters $\mathcal{K}$, or

$$\underline{u}_{k+1}(\tau) = \underline{u}_k(\tau) + \underline{f}_l\left(\underline{e}_k(\varsigma_1), \underline{e}_{k-1}(\varsigma_2), \mathcal{K}\right) \quad (3)$$

where $0 \le \varsigma, \varsigma_1, \varsigma_2 \le T$ and $\underline{e}_k(\tau) = \underline{r}(\tau) - \underline{y}_k(\tau)$ is the vector of output tracking errors recorded during whole current trial k. The reference $\underline{r}(t)$ is given and obviously it must be periodic with the same period T as controlled batch processes. The terms $\underline{f}_l\left(\underline{e}_k(\varsigma), \mathcal{K}\right)$ in (2) and $\underline{f}_l\left(\underline{e}_k(\varsigma_1), \underline{e}_{k-1}(\varsigma_2), \mathcal{K}\right)$ in (3) are often called parameters dependent first order and second order learning functions (or the update laws), respectively.

Essential research in the field of ILC focuses on how to determine effectively an appropriate learning function $\underline{f}_l\left(\underline{e}_k(\varsigma), \mathcal{K}\right)$ or $\underline{f}_l\left(\underline{e}_k(\varsigma_1), \underline{e}_{k-1}(\varsigma_2), \mathcal{K}\right)$ and their corresponding set of parameters $\mathcal{K}$, which

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guarantees the asymptotical convergence of output tracking error $\underline{e}_k(\tau) \to \underline{0}$ for all $0 \le \tau \le T$. Almost results obtained there in [2-12] are time discontinuous. It means that the inputs $\underline{u}_k(\tau)$ are updated just in $N = T/T_s$ steps during a trial with $\tau = iT_s$, $i = 0,1, \ldots, N-1$, where the time update period $0 < T_s \ll 1$ for system inputs is chosen arbitrarily small. Based on this time discrete update mode, both update laws (2) and (3) above are rewritten correspondingly in

$$\begin{aligned} \underline{u}_{k+1}(i) &= \underline{u}_k(i) + \underline{f}_l\left(\underline{e}_k(j), \mathcal{K}\right) \\ \underline{u}_{k+1}(i) &= \underline{u}_k(i) + \underline{f}_l\left(\underline{e}_k(j_1), \underline{e}_{k-1}(j_2), \mathcal{K}\right) \end{aligned} \tag{4}$$

where $0 \le j, j_1, j_2 \le N-1$.

In [2-10] the essential results of determination of convergent learning parameters $\mathcal{K}$ are presented, which are related mainly to the basic first order, time discrete linear learning functions of (4) as below

1) P-Type: $\underline{f}_l\left(\underline{e}_k(j), \mathcal{K}\right) = K\underline{e}_k(i)$, $\mathcal{K} = K$. (5)

2) D-Type: $\underline{f}_l\left(\underline{e}_k(j), \mathcal{K}\right) = K\underline{e}_k(i+1)$, $\mathcal{K} = K$. (6)

3) PD-Type: $\underline{f}_l\left(\underline{e}_k(j), \mathcal{K}\right) = K_1\underline{e}_k(i) + K_2\underline{e}_k(i+1)$, $\mathcal{K} = (K_1, K_2)$ (7)

4) PID-Type:K

$$\begin{aligned} &\underline{f}_l\left(\underline{e}_k(j), \mathcal{K}\right) = K_1\underline{e}_k(i-1) + K_2\underline{e}_k(i) + K_3\underline{e}_k(i+1) \\ &\mathcal{K} = (K_1, K_2, K_3) \end{aligned} \tag{8}$$

Although in ILC control concept for designing output feedback controller it is not required strictly to have a mathematical model of processes, but for determining effectively learning parameters $\mathcal{K}$ as well as for convergence analysis of it a model is obliged [2,3,7-9]. The consistent model is used for convergence analysis is the time discrete state-space model

$$\begin{cases} \underline{x}_k(i+1) = A\underline{x}_k(i) + B\underline{u}_k(i) \\ \underline{y}_k(i) = C\underline{x}_k(i) \end{cases} \tag{9}$$

For example, [2,3,5,10,11] show that by using D-Type learning function the parameter K there should satisfy

$$\|I - CB\mathcal{K}\| < 1. \tag{10}$$

The condition (10) is just sufficient. It means that a convergent parameter $\mathcal{K}$ may exist to ensure the requirement $\underline{e}_k(\tau) \to \underline{0}$, but it does not satisfy the aforementioned condition [12]. Moreover, in the case of $CB = \mathbf{0}$ (zeros matrix), none of $\mathcal{K}$ could be found from (10).

Furthermore, in spite that the method presented in [12] is model-free, it was established only for P-Type. Additionally, since all elements of $\mathcal{K} = (K_i)$ there are positive, it cannot be applied for proccesses, where the partial derivation of their input-output mapping changes the sign.

To overcome all these circumstances, the article will propose a few further conditions for determining convergence parameters $\mathcal{K}$ of elementary linear learning functions (5)-(8) above without using time discrete state space model (9) of linear processes. Precisely, these conditions are established based on transfer functions of process for both cases continuous time and discrete time. With these conditions the convergent parameters $\mathcal{K}$ could be still found, even in the situation of $CB = \mathbf{0}$.

The rest of this article is organized as follows. In Section 2 all theoretical substances related to transfer function based methods to determine convergent parameters $\mathcal{K}$ for linear learning functions are presented. Numerical simulations and discussions are shown in Section 3 to authenticate the performance of proposed approaches. Final Section provides conclusions and future works.

2. Main results

2.1. Overall Sufficient Condition

Consider a batch process described by input-output mapping as follows

$$\underline{u}_k(\tau) \mapsto \underline{y}_k(\tau) = \underline{f}_p\left(\underline{u}_k(\tau)\right) \tag{11}$$

with linear property

$$\underline{f}_p\left(A_1\underline{u}_k(\tau) + A_2\underline{v}_k(\tau)\right) = A_1\underline{f}_p\left(\underline{u}_k(\tau)\right) + A_2\underline{f}_p\left(\underline{v}_k(\tau)\right). \tag{12}$$

Then the application of first order update law given in (2) for this linear process yields

$$\begin{aligned} \underline{y}_{k+1}(\tau) &= \underline{f}_p\left(\underline{u}_{k+1}(\tau)\right) \\ &= \underline{f}_p\left(\underline{u}_k(\tau) + \underline{f}_l\left(\underline{e}_k(\varsigma), \mathcal{K}\right)\right) \\ &= \underline{f}_p\left(\underline{u}_k(\tau)\right) + \underline{f}_p\left(\underline{f}_l\left(\underline{e}_k(\varsigma), \mathcal{K}\right)\right) \\ &= \underline{f}_p\left(\underline{u}_k(\tau)\right) + \underline{f}_p \circ \underline{f}_l\left(\underline{e}_k(\varsigma), \mathcal{K}\right) \end{aligned}$$

Hence

$$\begin{aligned} \|\underline{e}_{k+1}(\tau)\| &= \|\underline{r}(\tau) - \underline{y}_{k+1}(\tau)\| \\ &= \left\|\underline{r}(\tau) - \underline{f}_p\left(\underline{u}_k(\tau)\right) - \underline{f}_p \circ \underline{f}_l\left(\underline{e}_k(\varsigma), \mathcal{K}\right)\right\| \\ &= \left\|\underline{r}(\tau) - \underline{y}_k(\tau) - \underline{f}_p \circ \underline{f}_l\left(\underline{e}_k(\varsigma), \mathcal{K}\right)\right\| \\ &= \left\|\underline{e}_k(\tau) - \underline{f}_p \circ \underline{f}_l\left(\underline{e}_k(\varsigma), \mathcal{K}\right)\right\| \\ &\le \left\|\mathbf{1}_e - \underline{f}_p \circ \underline{f}_l\left(\bullet, \mathcal{K}\right)\right\| \cdot \|\underline{e}_k(\tau)\| \end{aligned}$$

where $\mathbf{1}_e$ represents the identity mapping.

The last inequality shows that if the learning parameters $\mathcal{K}$ are chosen for satisfying the following sufficient condition

$$\left\| \mathbf{1}_e - (\underline{f}_p \circ \underline{f}_l)(\bullet, \mathcal{K}) \right\| < 1 \tag{13}$$

then we will obtain

$$\|\underline{e}_{k+1}(\tau)\| < \|\underline{e}_k(\tau)\|, \ \forall k$$

or the sequence of tracking errors will be decreased monotonously. Furthermore, by using the notion

$$\alpha = \sup_{\underline{e}(\varsigma)} \left\| \mathbf{1}_e - (\underline{f}_p \circ \underline{f}_l)(\underline{e}(\varsigma), \mathcal{K}) \right\|$$

it is attained $0 < \alpha < 1$ and therefore

$$\|\underline{e}_{k+1}(\tau)\| < \alpha^k \|\underline{e}_0(\tau)\|$$

which deduces the asymptotical convergence of

$$\lim_{k\to\infty} \|\underline{e}_k(\tau)\| = 0 .$$

The overall condition (13) is generally valid for non-linear update laws (4) and linear processes. But below it will be realized particularity in frequency domain for linear time continuous update laws by using their transfer function.

2.2. Realization for P-Type Learning Function

In following, we will apply the overall condition (13) presented above determining the parameter K of time continuous P-Type learning function

$$\underline{f}_l\left(\underline{e}_k(\tau), \mathcal{K}\right) = K \underline{e}_k(\tau) \tag{14}$$

to output tracking control a time continuous batch process described by the transfer function

$$G_p(s) = \frac{b_0 + b_1 s + \cdots + b_n s^n}{a_0 + a_1 s + \cdots + a_{n-1} s^{n-1} + s^n} . \tag{15}$$

In frequency domain the aforementioned P-Type learning function (14) is decribed by transfer function as below

$$G_l(s) = K . \tag{16}$$

The substitution of both (15) and (16) in the overall sufficient condition (13) yields

$$\left\| 1 - G_p(s) G_l(s) \right\| < 1$$

or

$$\left\| 1 - \frac{K\left(b_0 + b_1 s + \cdots + b_n s^n\right)}{a_0 + a_1 s + \cdots + a_{n-1} s^{n-1} + s^n} \right\| < 1 .$$

which is equivalent with

$$\left\|\left(a_0 - Kb_0, \ \ldots, a_{n-1} - Kb_{n-1}, 1 - Kb_n\right)\right\| \cdot \|\underline{\vartheta}_P\| < \left\|\left(a_0, \ \ldots, a_{n-1}, 1\right)\right\| \cdot \|\underline{\vartheta}_P\|$$

where

$$\underline{\vartheta}_P = \left(1, s, \ \ldots, s^n\right)^T . \tag{17}$$

Therefore, we come to the following deduction: *If the learning parameter K satisfies*

$$\left\|\left(a_0 - Kb_0, \ \ldots, a_{n-1} - Kb_{n-1}, 1 - Kb_n\right)\right\| < \left\|\left(a_0, \ \ldots, a_{n-1}, 1\right)\right\| \tag{18}$$

then the P-Type update law (14) *will provide for closed-loop systems the asymptotic convergence to zero of output tracking error*.

Finally, it is easily to see that the obtained sufficient condition (18) above still holds for time discrete linear processes described by following transfer function

$$G_p(z) = \frac{b_0 + b_1 z + \cdots + b_n z^n}{a_0 + a_1 z \ \cdots + a_{n-1} z^{n-1} + z^n} . \tag{19}$$

2.3. Realization for D-Type Learning Function

Time continuous learning function of D-Type

$$\underline{f}_l\left(\underline{e}_k(\tau), \mathcal{K}\right) = K \dot{\underline{e}}_k(\tau) \tag{20}$$

has following transfer function

$$G_l(s) = Ks . \tag{21}$$

Therefore, similarly as having done before with P-Type learning function (14), by applying the update law (20) to control the time continuous batch process (15) the overall sufficient condition (13) becomes

$$\left\| 1 - \frac{Ks\left(b_0 + b_1 s + \cdots + b_n s^n\right)}{a_0 + a_1 s + \cdots + a_{n-1} s^{n-1} + s^n} \right\| < 1 ,$$

which implies

$$\left\| a_0 + a_1 s + \cdots + a_{n-1} s^{n-1} + s^n - Ks\left(b_0 + b_1 s + \cdots + b_n s^n\right\} \right\| < \left\| a_0 + a_1 s + \cdots + a_{n-1} s^{n-1} + s^n \right\| .$$

Rewrite this inequality as matrix product

$$\left\|\left(a_0, a_1 - Kb_0, \ \ldots, a_{n-1} - Kb_{n-2}, 1 - Kb_{n-1}, -Kb_n\right) \underline{\vartheta}_D\right\| < \left\|\left(a_0, \ \ldots, a_{n-1}, 1, 0\right) \underline{\vartheta}_D\right\|$$

where $\underline{\vartheta}_D = \left(1, s, \ \ldots, s^n, s^{n+1}\right)^T$, then based on property of norm of matrix product it is obtained

$$\left\| \begin{pmatrix} a_0 \\ a_1 - Kb_0 \\ \vdots \\ a_{n-1} - Kb_{n-2} \\ 1 - Kb_{n-1} \\ -Kb_n \end{pmatrix} \right\| \cdot \left\| \underline{\vartheta}_D \right\| < \left\| \begin{pmatrix} a_0 \\ \vdots \\ a_{n-1} \\ 1 \\ 0 \end{pmatrix} \right\| \cdot \left\| \underline{\vartheta}_D \right\| ,$$

which is equivalent with

$$\left\| \left(a_0, a_1 - Kb_0, \ \dots \ , a_{n-1} - Kb_{n-2}, 1 - Kb_{n-1}, -Kb_n \right) \right\| < \left\| \left(a_0, \ \dots \ , a_{n-1}, 1, 0 \right) \right\|. \tag{22}$$

Hence we obtain: *If the learning parameter* K *of D-Type learning function* (20) *is chosen for satisfying the condition* (22), *then the output tracking errors* $\underline{e}_k(\tau)$ *of closed-loop system will converge to zero for all* τ. Evidently, this deduction holds also for time discrete processes described by (19).

2.4. Realization for PD-Type Learning Function

If the PD-Type learning function

$$\underline{f}_l \left(\underline{e}_k(\tau), \mathcal{K} \right) = K_1 \underline{e}_k(\tau) + K_2 \underline{\dot{e}}_k(\tau) \tag{23}$$

is applied to tracking control the time continuous process (15), then with transfer function of (23)

$$G_l(s) = K_1 + K_2 s$$

the sufficient condition (13) becomes

$$\left\| a_0 + a_1 s + \ \cdots \ + a_{n-1} s^{n-1} + s^n - \left(K_1 + K_2 s \right) \left(b_0 + b_1 s + \ \cdots \ + b_n s^n \right\} \right\| < \left\| a_0 + a_1 s + \ \cdots \ + a_{n-1} s^{n-1} + s^n \right\|$$

what equivalent with

$$\left\| \left(a_0 - K_1 b_0, a_1 - K_1 b_1 - K_2 b_0, \ \dots \ , a_{n-1} - K_1 b_{n-1} - K_2 b_{n-2}, 1 - K_1 b_n - K_2 b_{n-1}, -K_2 b_n \right) \underline{\vartheta}_{PD} \right\| < \left\| \left(a_0, \ \dots \ , a_{n-1}, 1, 0 \right) \underline{\vartheta}_{PD} \right\|$$

where $\underline{\vartheta}_{PD} = \underline{\vartheta}_D = \left(1, s, \ \dots \ , s^n, s^{n+1} \right)^T$. Hence, we have

$$\left\| \begin{pmatrix} a_0 - K_1 b_0 \\ a_1 - K_1 b_1 - K_2 b_0 \\ \vdots \\ a_{n-1} - K_1 b_{n-1} - K_2 b_{n-2} \\ 1 - K_1 b_n - K_2 b_{n-1} \\ -K_2 b_n \end{pmatrix} \right\| < \left\| \begin{pmatrix} a_0 \\ \vdots \\ a_{n-1} \\ 1 \\ 0 \end{pmatrix} \right\| . \tag{24}$$

Consequently, it conducts to:

If the set of learning parameters $\mathcal{K} = (K_1, K_2)$ *of applied PD-Type learning function* (23) *satisfies the condition* (24), *then the output tracking errors* $\underline{e}_k(\tau)$ *of closed-loop system converges asymptotically to zero*.

Obviously the condition (24) also holds for time discrete linear processes described in frequency domain by transfer function (19).

2.5. Realization for PID-Type Learning Function

The transfer function of time continuous PID-Type learning function

$$\underline{f}_l \left(\underline{e}_k(\tau), \mathcal{K} \right) = = K_1 \int_0^\tau \underline{e}_k(\zeta) d\varsigma + K_2 \underline{e}_k(\tau) + K_3 \underline{\dot{e}}_k(\tau) \tag{25}$$

is as follows

$$G_l(s) = \frac{K_1 + K_2 s + K_3 s^2}{s} .$$

Accordingly, the overall sufficient condition (13) will be expressed as

$$\left\| 1 - \frac{\left(K_1 + K_2 s + K_3 s^2 \right) \left(b_0 + b_1 s + \ \cdots \ + b_n s^n \right)}{s \left(a_0 + a_1 s + \ \cdots \ + a_{n-1} s^{n-1} + s^n \right)} \right\| < 1$$

or

$$\left\| s \left(a_0 + a_1 s + \ \cdots \ + a_{n-1} s^{n-1} + s^n \right) - \left(K_1 + K_2 s + K_3 s^2 \right) \left(b_0 + b_1 s + \ \cdots \ + b_n s^n \right) \right\| < \left\| s \left(a_0 + a_1 s + \ \cdots \ + a_{n-1} s^{n-1} + s^n \right) \right\| ,$$

what is equivalent with

$$\left\| \left(-K_1 b_0, a_0 - K_1 b_1 - K_2 b_0, a_1 - K_1 b_2 - K_2 b_1 - K_3 b_0, \ \dots \ , a_{n-1} - K_1 b_{n-1} - K_2 b_{n-2} - K_3 b_{n-3}, 1 - K_1 b_n - K_2 b_{n-1} - K_3 b_{n-2} \right) \underline{\vartheta}_{PID} \right\| < \left\| \left(0, 0, a_0, \ \dots \ , a_{n-1}, 1 \right) \underline{\vartheta}_{PID} \right\|$$

where $\underline{\vartheta}_{PID} = \left(1, s, \ \dots \ , s^{n+1}, s^{n+2} \right)^T$. Therefore

$$\left\| \begin{pmatrix} -K_1 b_0 \\ a_0 - K_1 b_1 - K_2 b_0 \\ \vdots \\ a_{n-1} - K_1 b_n - K_2 b_{n-1} - K_3 b_{n-2} \\ 1 - K_2 b_n - K_3 b_{n-1} \\ -K_3 b_n \end{pmatrix} \right\| < \left\| \begin{pmatrix} 0 \\ a_0 \\ \vdots \\ a_{n-1} \\ 1 \\ 0 \end{pmatrix} \right\| . \tag{26}$$

Finally, *if the set* $\mathcal{K} = (K_1, K_2, K_3)$ *of PID-Type update law* (25) *meets the sufficient condition given in* (26) *then the iterative learning controller will cause the output of closed-loop system asymptotically to desired reference* $r(t)$. This deduction holds also for time discrete processes described by (19).

2.6. Usage Opportunity with State-Space Model

As mentioned before, linear processes described primarily in time domain

$$\dot{\underline{x}} = A\underline{x} + Bu\,,\ \ y = C\underline{x} \qquad (27)$$

where $CB = 0$ it is impossible to determine learning parameters by using condition related with this time domain model, such $\|1 - CB\mathcal{K}\| < 1$ presented in [3].

In this circumstance we can use the proposed conditions (18),(22),(24) or (26) in frequency domain for this purpose. To do that, first we convert the model (27) in frequency domain with

$$G_p(s) = C(sI - A)^{-1}B$$

and then determine the learning parameter set $\mathcal{K}$ by using coefficients a_i, b_j of $G_p(s)$ accordingly to (18) if applied update law is P-Type, or (22) when applied update law is D-Type, or (24) for the application of PD- update law, or (26) by applying PID- update law.

2.7. Application to MIMO Processes

Consider a MIMO linear, time invariant batch process with n inputs and m output. In frequency domain this process is described by a transfer matrix $\mathbf{G}_p(s)$ of dimension $m \times n$ as below:

$$\mathbf{G}_p(s) = \begin{pmatrix} G_{11}(s) & \cdots & G_{1n}(s) \\ \vdots & \ddots & \vdots \\ G_{m1}(s) & \cdots & G_{mn}(s) \end{pmatrix}.$$

Denote the transfer matrix of applied linear learning functions $\underline{f}_l\left(\underline{e}_k(\varsigma), \mathcal{K}\right)$ with $\mathbf{G}_l(s, \mathcal{K})$, then this matrix is of dimension $n \times m$. For the scenario that this chosen learning function makes the matrices product $\mathbf{G}_p(s)\mathbf{G}_l(s, \mathcal{K})$ becomes diagonal, then the set $\mathcal{K}$ of learning parameters would be determined in manner that all diagonal entries of it satisfy sufficient conditions (18), (22), (24) or (26).

2.8. Integrating Proposed Conditions in ILC Control Algorithm

In order to facilitate the integration of proposed sufficient conditions in an intelligent controller created with ILC concept the following control algorithm is established. In this algorithm it is assumed that the repetitive time T, if controlled process is continuous time, or repetitive steps N, if the process is discrete time, are known.

Each `while-loop` in aforementioned algorithm represents a trial. In addition, the set $\mathcal{K}$ of learning parameters may be determined by solving compatibly an optimization problem with constraints, instead of choosing it directly via inequalities (18),(22),(24) or (26). Such a problem could be as below

$$\mathcal{K}^* = \arg\min_{\mathcal{K}}\left(R - L(\mathcal{K})\right) \text{ subject to } R > L(\mathcal{K}).$$

where $R,\ L(\mathcal{K})$ are right and left site of (18),(22), (24) and (26), respectively.

`Algorithm`: ILC control algorithm with learning parameters determined with proposed condition

1	Approximate a transfer function with coefficients a_i, b_j for controlled process. Choose $0 < T_s \ll 1$ if the process is continuous time, then calculate $N = T/T_s$. Choose an interesting update law within four existing P-, D-, PD- and PID-Type. Determine the set $\mathcal{K}$ of learning parameters from a_i, b_j based on proposed conditions via (18) if chosen update law is P-Type, or (22) for D-Type update law, or (24) if update law is PD-Type, or (26) for chosen update law PID-Type. Set $u(i) = r(i),\ e(i) = 0,\ i = 0,1,\ \ldots\ ,N-1$.
2	**`while`** continue the control **`do`**
3	**`for`** $i = 0,1,\ \ldots\ ,N-1$ **`do`**
4	Send $u(i)$ to process for a while of T_s. Measure the output $y(i)$. Calculate $e(i) = r(i) - y(i)$. Determine $u(i) = u(i) + Ke(i)$ for P-Type or $u(i) = u(i) + Ke(i+1)$ for D-Type or $u(i) = u(i) + K_1e(i) + K_2e(i+1)$ for PD-Type or $u(i) = u(i) + K_1e(i-1) + K_2e(i) + K_3e(i+1)$ if PID-Type learning function is applied.
5	**`end for`**
6	**`end while`**

3. Numerical Examples

To verify the application ability of proposed sufficient conditions established in frequency domain, in following some examples will be executed per numerical simulations.

3.1. Simulation 1: Control with P-Type Learning Function for Continuous Time Process

Consider a time continuous, linear time invariant process in frequency domain as below:

$$G_p(s) = \frac{1+s}{6+5s+s^2}\,. \qquad (28)$$

This transfer function has following model parameters $a_0 = 6,\ a_1 = 5$ and $b_0 = b_1 = 1,\ b_2 = 0$.

For using the P-Type learning function to control the process (28) the learning parameter $K = 5$ for this P-Type update law is chosen. This learning parameter satisfies the condition (18), because

$$\left\|\left(a_0 - b_0K\ ,\ a_1 - b_1K\ ,\ 1 - b_2K\right)\right\| = \left\|\left(1\ ,\ 0\ ,\ 1\right)\right\| = 1.4142$$

and

$$\left\|\left(a_0\ ,\ a_1\ ,\ 1\right)\right\| = \left\|\left(6\ ,\ 5\ ,\ 1\right)\right\| = 7.8740\,.$$

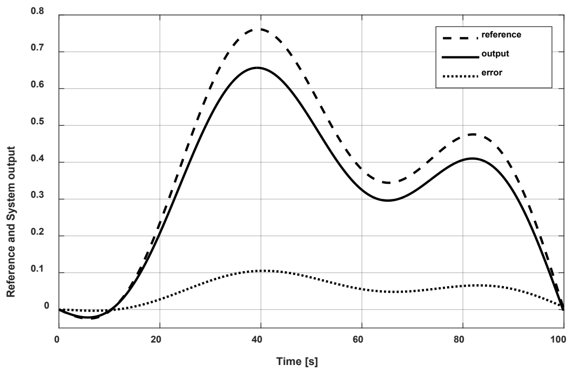

Fig. 1. Output tracking results after 2 trials.

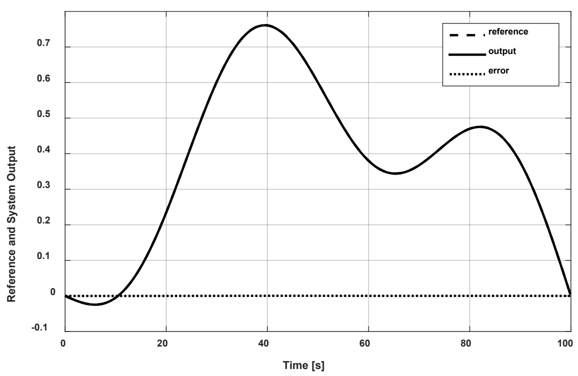

Fig. 2. Output tracking results after 5 trials.

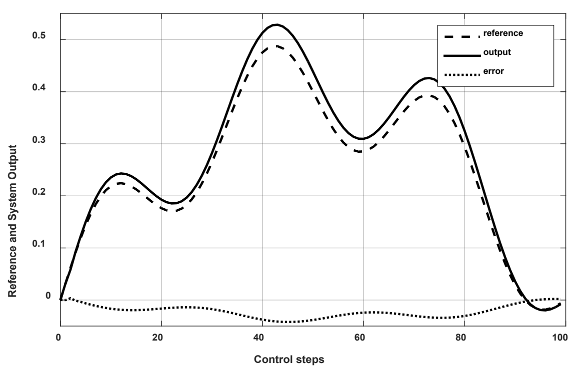

Fig. 3. Output tracking results after 3 trials.

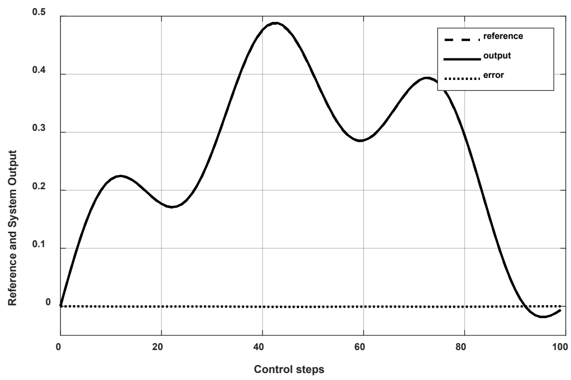

Fig. 4. Output tracking results after 10 trials.

Suppose the process is repetitive with $T = 100s$, then with sufficiently small $T_s = 0.1s$ we obtain with the desired reference

$$r(t) = 0.6\sin\left(\pi t/T\right) - 0.2\sin\left(4\pi t/T\right)$$

the simulation results as exhibited in Fig. 1 and Fig. 2. In the simulation, the model (28) is just used to declare the process dynamic and it was carried out through its equivalent state space model.

As seen there, just after 5 trials the system output had reached perfectly reference $r(t)$ as expected. Hence, the simulation results had proved the rightness of the proposed sufficient condition given in (18).

Note that for process (28) the learning parameter $K = 5$ cannot be determined with time domain condition (10).

3.2. Simulation 2: Control with D-Type Learning Function for Discrete Time Process

The simulation hereafter is carried out for time discrete system described by

$$G_p(z) = \frac{0.5 + z}{0.1 + 0.7z + z^2} \tag{29}$$

by applying D-Type update law.

For the D-Type update law (20) we choose the learning parameter $K = 0.5$, because it satisfies the established condition (22), with

$$\left\|\left(a_0, a_1, 1, 0\right)\right\| = \left\|\left(0.1\ ,\ 0.7\ ,\ 1\ ,\ 0\right)\right\| = 1.2247\ .$$

and

$$\left\|\left(a_0, a_1 - b_0K, 1 - b_1K, -b_2K\right)\right\| = \left\|\left(0.1\ ,\ 0.45\ ,\ 0.5\ ,\ 0\right)\right\|$$
$$= 0.6801$$

Furthermore, we can see that this learning parameter $K = 0.5$ cannot be determined by using time domain condition given in (10).

Both Fig. 3 and Fig. 4 illustrated output tracking performance of closed-loop system after 3 and 10 trials, respectively. In the simulation are assigned

$$N = 100$$

for the number of control steps during a working period of process, and

$$r(i) = 0.4\sin\frac{\pi i}{N} + 0.1\sin\frac{6\pi i}{N}\ ,$$

with $i = 0, 1,\ \ldots\ , N-1$ for the references.

Moreover, to implement the process dynamic, the system transfer function (29) had been converted correspondingly in time domain with its difference equation as below:

$$y_k(i) + 0.7y_k(i-1) + 0.1y_k(i-2) =$$
$$= u_k(i-1) + 0.5u_k(i-2) .$$

Again, these obtained simulation results had confirmed deeply the applicable ability of established condition (22) for determining a convergent learning parameter K . With the learning parameter, which is chosen accordingly to(22), the system output tends rush to the desired reference $r(t)$. Particularity, the system output is coincided with its reference just after 10 trials, which was demonstrated in Fig. 4.

3.3. Simulation 3: Control with PD-Type Learning Function for Continuous Time Process

Consider a third order linear time invariant and time continuous process described primarily by

$$\begin{cases} \dot{\underline{x}} = A\underline{x} + Bu \\ y = C\underline{x} \end{cases} \tag{30}$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{pmatrix},\ B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},\ C = \begin{pmatrix} 1 , & 1 , & 0 \end{pmatrix}$$

Since there $CB = 0$ a convergent learning parameter set $\mathcal{K}$ cannot be determined by using a conventional condition in time domain, such as by using the condition $\|1 - CBK\| < 1$. In this situation we will apply the proposed conditions in frequency domain for determining learning parameters set $\mathcal{K}$.

For determining learning parameters set $\mathcal{K}$ with proposed conditions in frequency domain, first the process (30) will be rewritten equivalently in frequency domain with the transfer function

$$G_p(s) = C(sI - A)^{-1}B = \frac{1+s}{24 + 26s + 9s^2 + s^3} \tag{31}$$

then we use it for determining $\mathcal{K}$

In comparison with (15) the coefficients of this transfer function are respectively as below

$$a_0 = 24,\ a_1 = 26,\ a_2 = 9$$

and $b_0 = 1,\ b_1 = 1,\ b_2 = b_3 = 0$

In following the iterative learning controller with PD-Type learning function (23) is applied to tracking control for a time continuous process (30) to a desired reference $r(t)$. To do that, the equivalent transfer function (31) will be used for determining learning parameters set $\mathcal{K} = (K_1, K_2)$.

It can be seen that the chosen set of

$$K_1 = 12,\ K_2 = 7$$

satisfies sufficiently the requirement of condition (24) in frequency domain, with

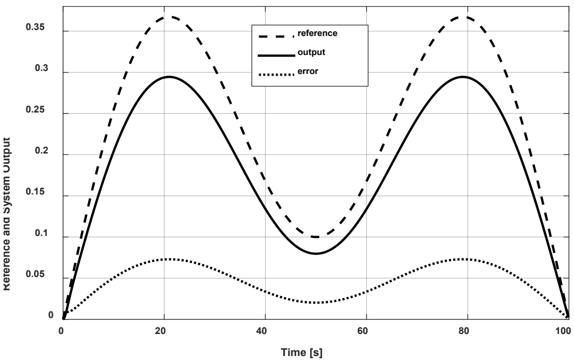

Fig. 5. Output tracking results after 2 trials.

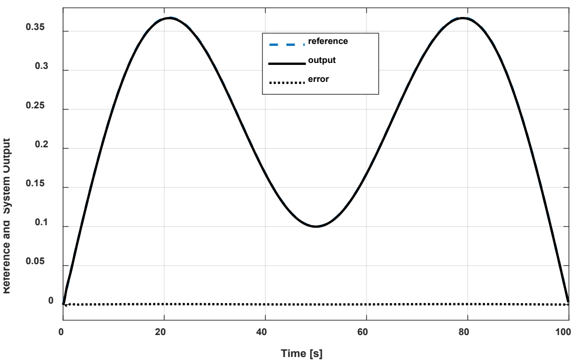

Fig. 6. Output tracking results after 5 trials.

$$\left\| \begin{pmatrix} a_0 - K_1 b_0 \\ a_1 - K_1 b_1 - K_2 b_0 \\ a_2 - K_1 b_2 - K_2 b_1 \\ 1 - K_1 b_3 - K_2 b_2 \\ -K_2 b_3 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 12 \\ 7 \\ 2 \\ 1 \\ 0 \end{pmatrix} \right\| = 14.0712$$

and

$$\left\| \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ 1 \\ 0 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 24 \\ 26 \\ 9 \\ 1 \\ 0 \end{pmatrix} \right\| = 35.5240$$

Fig. 5 and Fig. 6 exhibit simulation results with chosen learning parameters $\mathcal{K} = (K_1, K_2)$ above and

$$T = 100s,\ T_s = 0.1$$

$$r(t) = 0.3\sin\frac{\pi t}{T} + 0.2\sin\frac{3\pi t}{T} .$$

These obtained simulation results confirmed once more the application ability of proposed condition for the circumstance $CB = 0$, that the sufficient condition (24), which was established in frequency domain, had provided convergence learning parameter set $\mathcal{K} = (K_1, K_2)$ for closed-loop system as expected. Particularly, just after 5 trials both system output and the reference are almost overlapped to each other with $e_k(\tau) \approx 0$ for all repetitive time interval $0 \le \tau \le T$.

3.4. Simulation 4: Control with PID-Type Learning Function for Discrete Time Process

We consider the output tracking control problem for time discrete linear process described primarily in time domain

$$\begin{cases} \underline{z}_k(i+1) = \begin{pmatrix} 0 & 1 \\ -0.2 & -0.5 \end{pmatrix} \underline{z}_k(i) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_k(i) \\ y_k(i) = \left(0.3 \ , \ 1\right) \underline{z}_k(i) \end{cases} \tag{32}$$

which is repetitive with $N = 100$, so that the process output converges asymptotically to desired reference $r(t)$ in trapezoid format as below

$$r(i) = \begin{cases} 1 & \text{if } \ 20 \le i \le 80 \\ i/20 & \text{if } \ i \le 20 \\ r(100-i) & \text{if } \ 80 \le i \end{cases} \tag{33}$$

for all indexes

$$i = 0, 1, \ \dots \ , N-1 .$$

In following we will solve this control problem by using ILC concept with PID-Type update law. To determine the learning parameter set K_1, K_2, K_3 for applied PID learning function (25) accordingly to the sufficient condition (26) we convert equivalently first the state-space model above in frequency domain

$$G_p(z) = C(zI - A)^{-1} B = \frac{0.3 + z}{0.2 + 0.5z + z^2}$$

and obtain

$$a_0 = 0.3, \ a_1 = 1,$$
$$b_0 = 0.2, \ b_1 = 0.5, \ b_2 = 0.$$

Based on these coefficients and according to the condition (26) we choose

$$K_1 = 0.1, \ K_2 = 0.15 \ \text{ and } \ K_3 = 0.2 ,$$

because they satisfy the requirement of (26) with

$$\begin{aligned} &\left\| \left(-K_1 b_0 \ , \ a_0 - K_1 b_1 - K_2 b_0, \right.\right. \\ &\quad \left.\left. a_1 - K_2 b_1 - K_3 b_0 \ , \ 1 - K_3 b_2 \right) \right\| = \\ &\quad\quad = \left\| \left(-0.045 \ , \ -0.04 \ , \ 0.14 \ , \ 1\right) \right\| = 1.0115 \end{aligned}$$

and

$$\left\| \left(0 \ , a_0 \ , a_1 \ , 1\right) \right\| = \left\| \left(0 \ , 0.3 \ , 1 \ , 1\right) \right\| = 1.4457 .$$

Fig. 7 and Fig. 8 demonstrated simulation results after 3 and 10 trials, respectively. These obtained simulation results authenticated once more the expected applicability of established condition (26), that with learning parameters being chosen accordingly to this condition, the system output converges really to desired reference just after 10 trials, with a small average tracking error as given below:

$$\left\| \underline{e}_{10}(i), \ i = \overline{1, N} \right\| \approx 0.0612 .$$

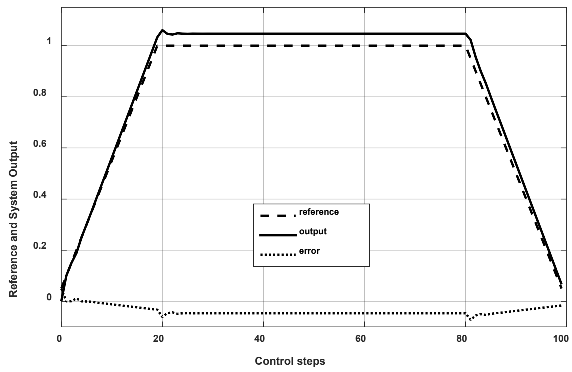

Fig. 7. Output tracking results after 3 trials.

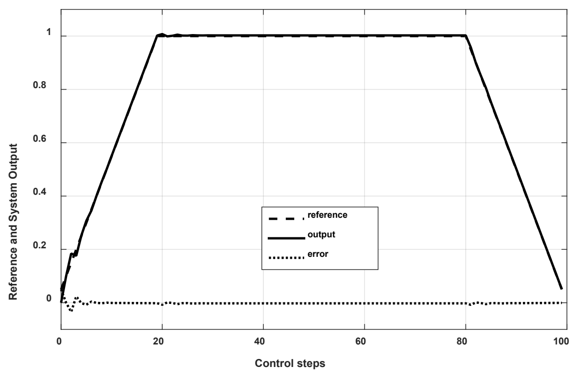

Fig. 8. Output tracking results after 10 trials.

4. Conclusion

Several sufficient conditions in frequency domain for determining convergence learning parameters in ILC concept had been presented in the paper. With these conditions many convergence parameters can be found out, what may be impossible with existing conditions created in time domain. This affirmation had been authenticated also theoretically and per simulation in the article.

Furthermore, these given frequency conditions had been realized in detail for four elemental linear update laws, including P-, D-, PD-, PID- Type, so that they become convenient in application. Few numerical simulations having carried out with **`m.files`** in MatLab had confirmed the rightness of this assertion.

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